Evolution off the Main Sequence

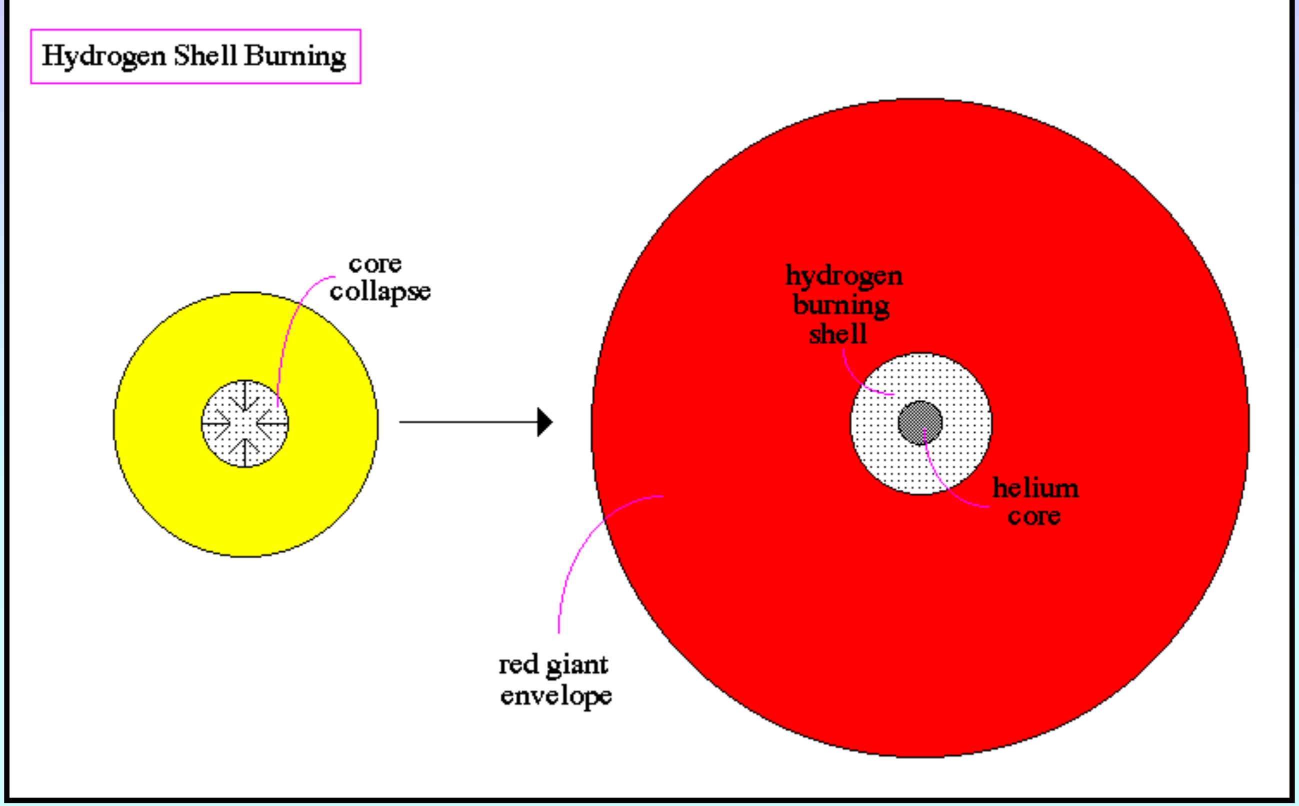
Zero Age Main Sequence (ZAMS) stars are defined as being chemically homogeneous. However, because (most) stars are not fully convective, nuclear fusion in the stellar cores will create chemical gradients. In particular, the fraction of hydrogen in the cores will decrease, as hydrogen is fused into helium.

When the hydrogen fraction in the center of the star declines to ~5%, the main sequence phase ends.

Evolution to the Giant Branch

Roughly speaking, a star's main sequence hydrogen-to-helium burning core consists of the inner ~10% (by mass) of the star. When all the core hydrogen is used up

- The core begins to collapse, as there is no longer an energy source to maintain its central pressure. The core's gravitational potential energy is converted into thermal energy.
- The core collapse also increases the pressure just outside the core, causing hydrogen fusion in a thick shell around the core (~5% of the star). This shell will eventually get very thin (~ 0.5% by mass).
- As conditions in the shell become more extreme, fusion proceeds via CNO burning, with the rate entirely a function of the core mass, i.e., $L \propto M^{\gamma}$, with $\gamma \sim 8$.
- The radiation pressure associated with shell burning pushes matter away in both directions.
- Quick rule of thumb: shells inverse expansion. If the core is contracting, the material outside a shell is expanding.



The post main-sequence star has 2 energy sources: the gravitational contraction of the core, and the hydrogen fusion occurring in the shell around the core. The amount of material fusing is small, but the rate of fusion is high.

During the giant branch, the evolutionary rate constantly increases: shell fusion produces helium which becomes part of the core, which increases the mass of the core, which increases the rate of fusion.

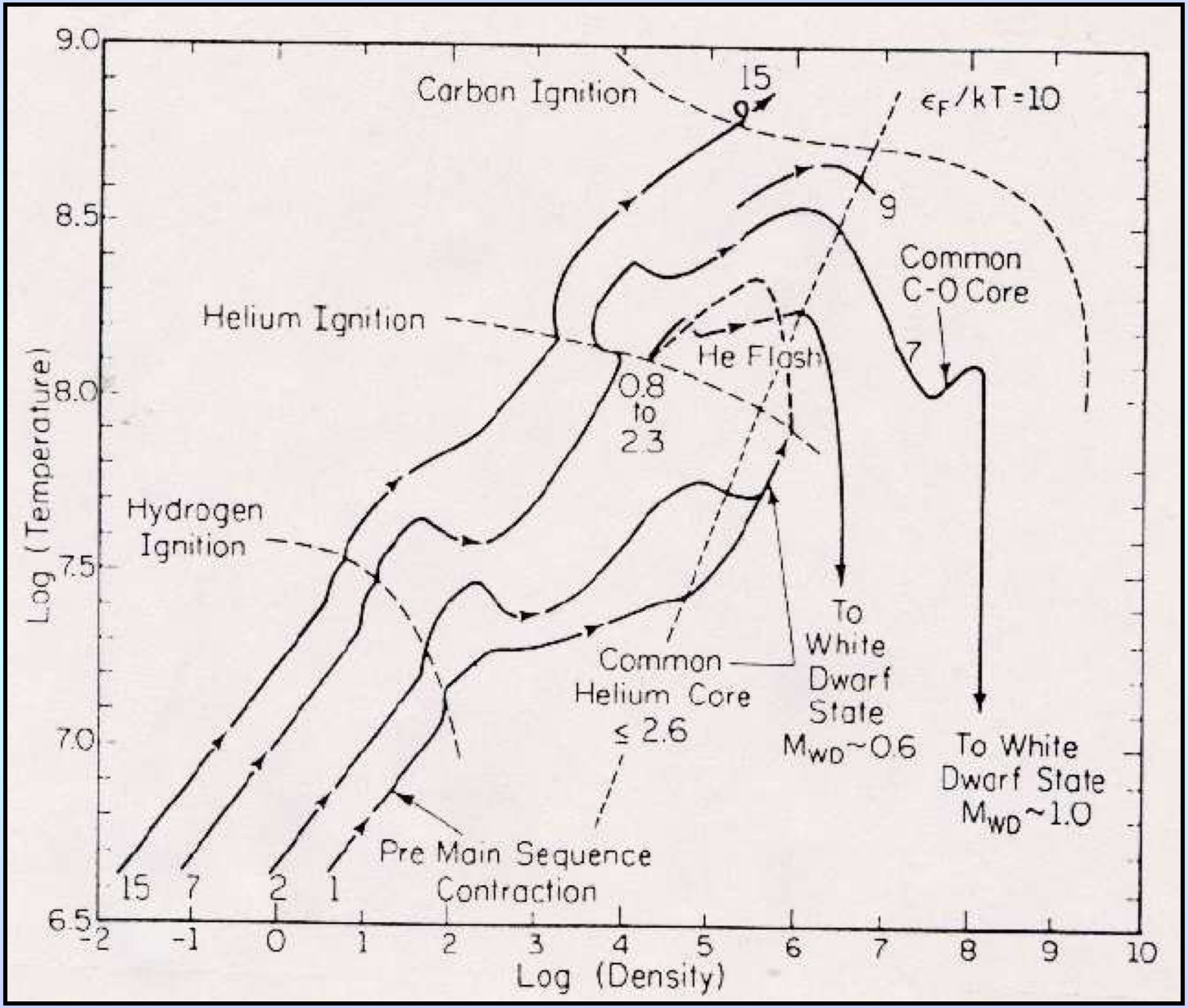
Evolution to the Giant Branch

- As the shell narrows, the star adjusts its structure on a thermal timescale. As the area outside the shell is pushed farther away by radiation pressure, it begins to cool. The area becomes more opaque because of Kramer's opacity law ($\kappa \propto T^{-3.5}$).
- The higher opacity of the envelope traps the energy, which does PdV work on its surroundings, causing the gas to expand. This further cools the gas, which increases the opacity, which traps more energy, which causes further expansion. The star crosses the "Hertzprung Gap" (the region between the main sequence and the red giant branch in the HR diagram), with the expansion driven mostly by its own thermal energy.
- As the outside of the star cools, conditions become more conducive to convection (again, since $\kappa \propto T^{-3.5}$). The convective envelope reaches deeper and deeper into the star. Eventually, the star is almost fully convective (except for the inert core and the very thin hydrogen burning shell).

Evolution on the Giant Branch

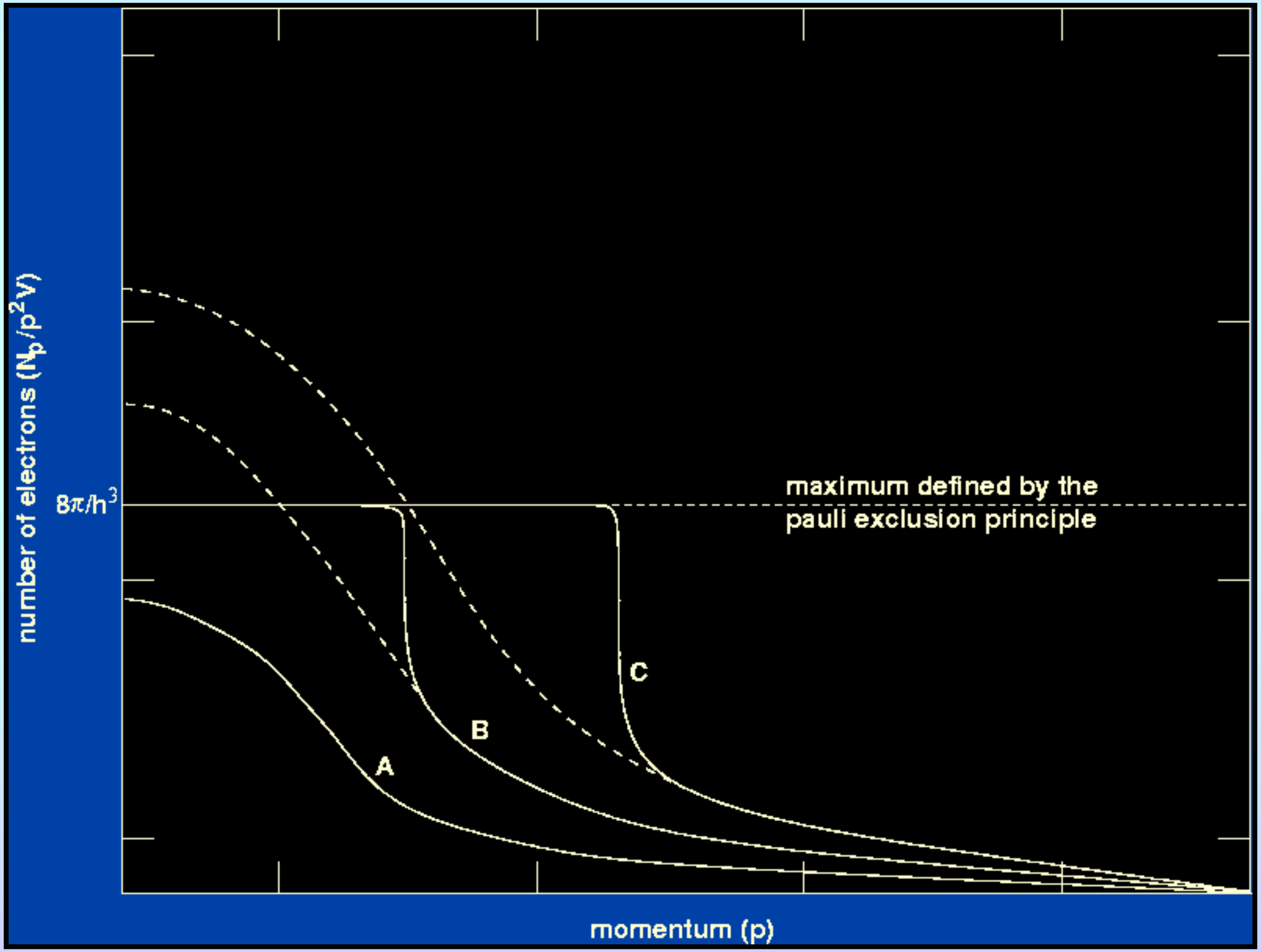
- During their main sequence phase, high-mass stars have cores that become smaller with time. When the stars become giants, their convective envelope can reach the outer region of the early-main sequence core, and "dredge-up" some CNO-processed material.
- Eventually, the star reaches the "Hayashi line." (Energy-producing stars cannot be cooler than this line.)
- Kramer's-style opacity dominates throughout most of the star, but at the surface of cool stars, H^- is the dominant source of opacity. The requisite electrons (for bound free and H^-) come from metals; consequently, the lower the star's metal abundance, the lower the opacity, the less energy is trapped, the less PdV work is done, the smaller the stellar expansion, and the hotter the star. The location of the Hayashi line is temperature dependent: metal-poor giants are bluer than metal-rich giants.

Evolution of the Central Conditions



Electron Degeneracy

At high densities, the Maxwellian distribution comes up against the Pauli exclusion principle, $dV\, dp = (\Delta x\ \Delta p_x)(\Delta y\ \Delta p_y)(\Delta z\ \Delta p_z) \sim h^3$. This forces electrons to high momentum states, increasing the pressure.



The Triple-α Process

- Helium is inert because ^{8}Be exists for only ~ 10^{-16} seconds. However, eventually, the core becomes dense enough so that the reaction $^{4}He + {}^{4}He \leftrightarrows {}^{8}Be + {}^{4}He \rightarrow {}^{12}C^*$ becomes possible. Unfortunately, the decay of excited carbon, $^{12}C^* \rightarrow {}^{12}C$ is highly unlikely; much more likely is $^{12}C^* \rightarrow {}^{4}He + {}^{8}Be$. Still, once the $^{4}He + {}^{4}He \rightarrow {}^{8}Be + {}^{4}He \rightarrow {}^{12}C^* \rightarrow {}^{12}C$ reaction starts, its energy quickly causes more reactions, creating a runaway, with $\varepsilon_{3\alpha} \propto T^{40}$. This is called the triple-α process.
- Due to the effects of electron degeneracy, stars with $M < 2.3\ M_\odot$ all ignite helium when the core mass reaches ~ $0.45\ M_\odot$.
- With triple-α also comes the reaction $^{12}C + {}^{4}He \rightarrow {}^{16}O$. But the resonance for this reaction is not well-known, so the resulting ratio of $^{12}C/^{16}O$ can be almost anything. (Consequently, it is usually assumed to be 50%-50%.) Although the electrostatic repulsion is greater, the $^{16}O + {}^{4}He \rightarrow {}^{20}Ne$ reaction can also occur.

The Helium Flash

- When helium ignites in low mass stars ($M < 3\ M_\odot$), it does so degenerately in a thermonuclear runaway, called the "Helium Flash". At maximum, the luminosity from this fusion is $10^{11}\ L_\odot$, like a supernovae! However, almost none of this energy reaches the surface; it all goes into lifting the core degeneracy and then heating up the star. As a result, the core expands, hydrogen shell burning ceases (or becomes very small), the stellar opacity declines, less heat is trapped, and the star becomes smaller and fainter.
- In higher mass stars, helium ignites non-degenerately. In very high mass stars ($M > 15\ M_\odot$), it may even ignite before the star gets to the giant branch. (In these stars, the nuclear timescale is similar to the thermal timescale.)

Mass Loss

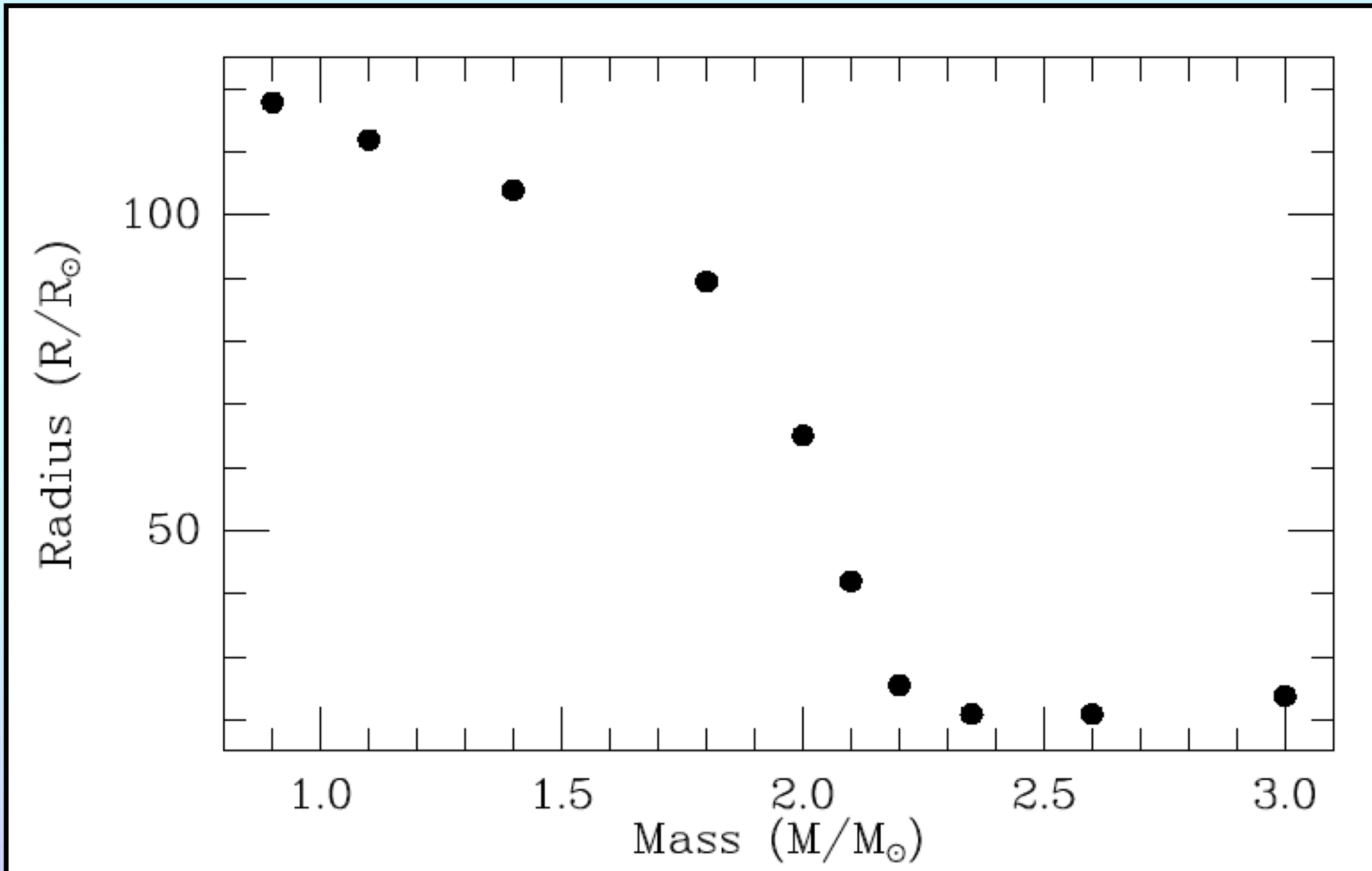
• On the giant branch (and especially on the return to the giant branch), mass loss becomes important. A “reasonable” mass-loss law is

$$\dot{M} \propto \frac{L\,R}{M}$$

(In other words, the mass lost from a star is proportional to the luminosity blowing the material away, and inversely proportional to the gravitational potential at the stellar surface.) The constant of proportionality for the equation is defined by the Sun, which loses roughly $4 \times 10^{-13}\, M_{\odot}$/yr (the solar wind). This is the *Reimers’* mass-loss law. It is almost certainly wrong, but in some cases, it is perhaps not too far wrong. During the red giant branch, mass loss occurs, but is relatively small; in total, about $\sim 0.2\, M_{\odot}$ is lost.

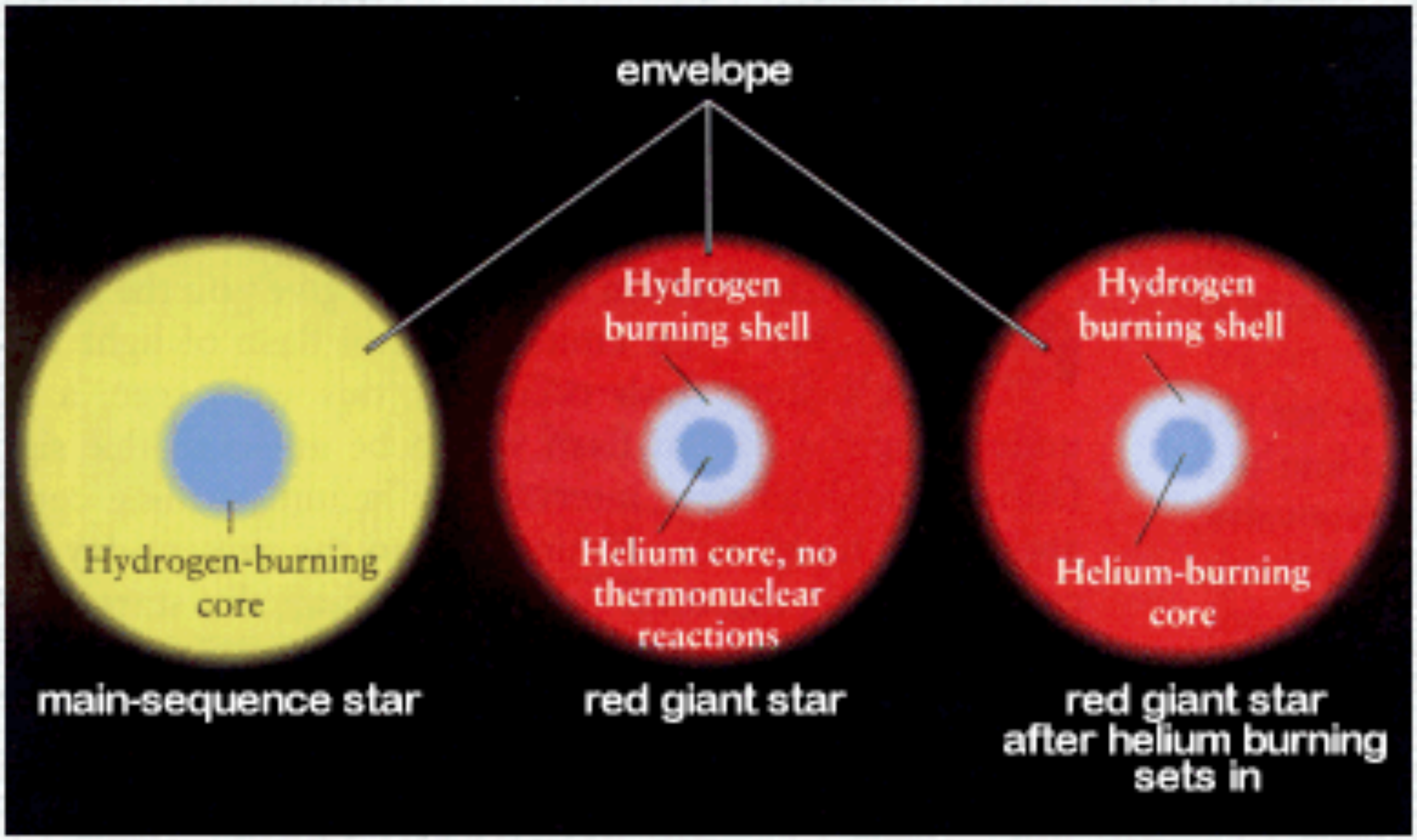
Stellar Sizes on the Red Giant Branch

• The maximum size a star attains before igniting helium depends on its mass. Electron degeneracy helps support the core of lower masses stars, delaying the ignition. As a result, lower mass stars get larger than higher mass stars.



Horizontal Branch Stars

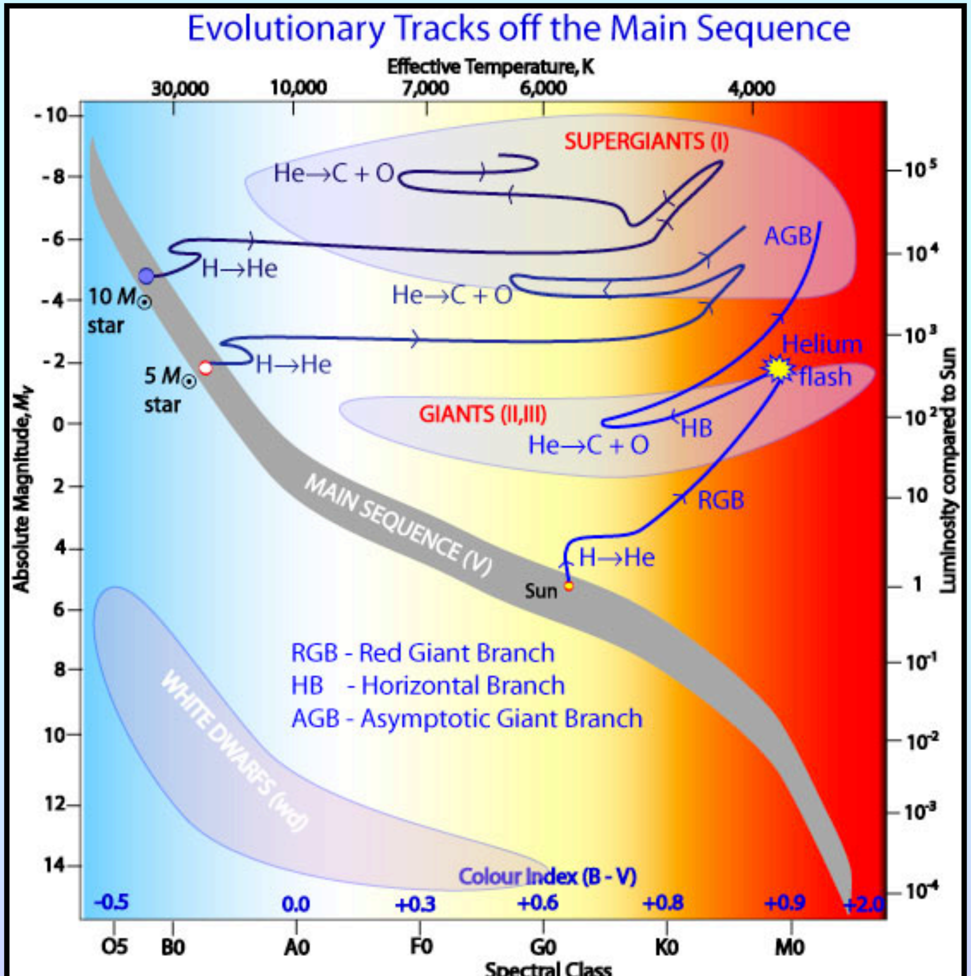
• After helium ignition, helium fusion occurs in the center of the (previously inert) helium core. Hydrogen continues to fuse in a shell outside the core.

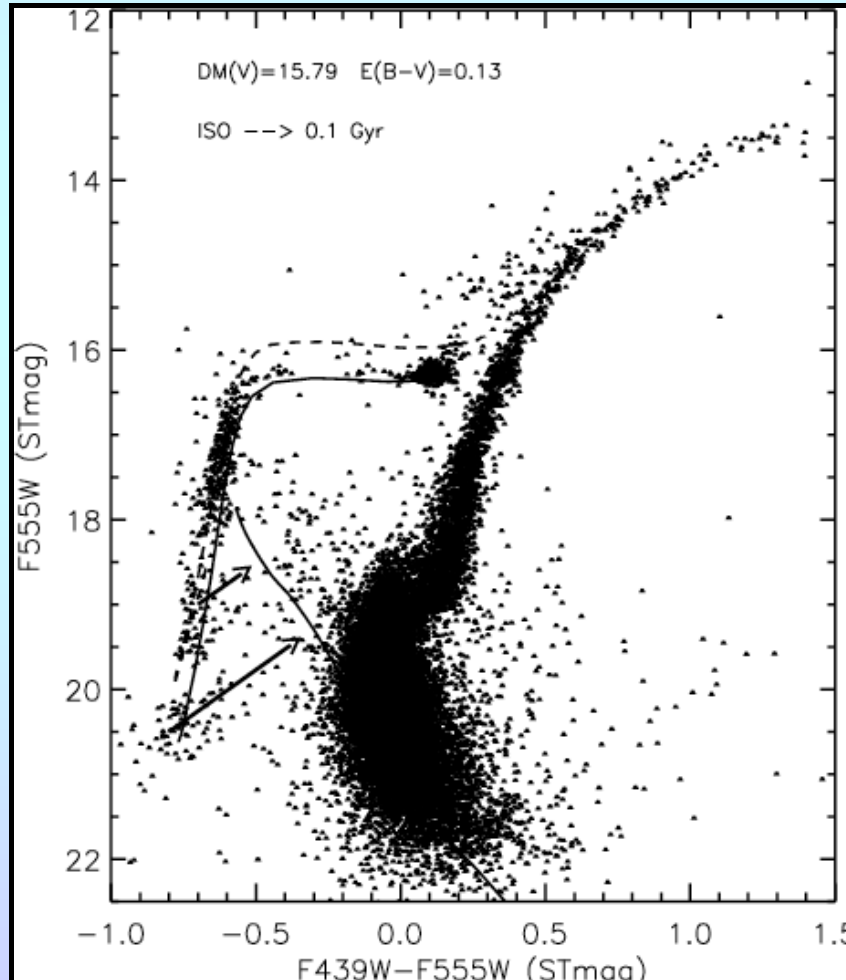


As a star evolves, thermonuclear reactions occur at different locations within its interior.

Horizontal Branch Stars

• Low mass stars ($M \lesssim 1\ M_\odot$) which ignite helium degenerately settle onto the "Horizontal branch," with luminosities of $\sim 75\ L_\odot$. The temperature of these stars depend on their envelope mass: the more matter on top of the hydrogen burning shell, the cooler the star.





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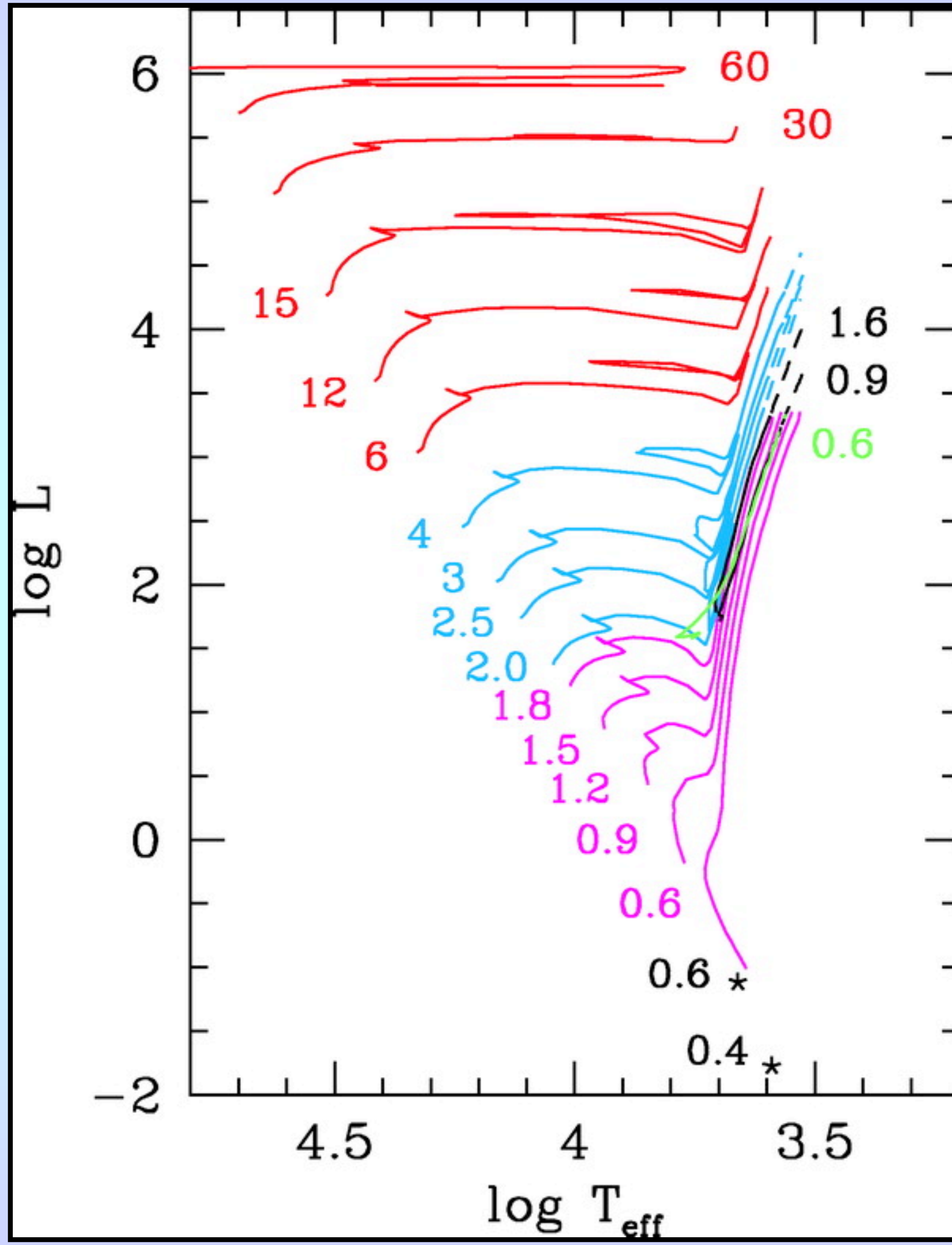
• The bluest horizontal branch stars have masses $\sim 0.5\ M_\odot$; the reddest, $\sim 1\ M_\odot$.

• Some horizontal stars have very small envelopes ($\sim 0.05\ M_\odot$), and thus extremely blue colors. These are sometimes called extreme horizontal branch stars (EHB), and appear on the "vertical part of the horizontal branch".

• Helium fusion is much less efficient than hydrogen fusion, so the horizontal branch phase is relatively short ($\lesssim$ Gyr). When helium in the core runs out, the core again contracts, and a thick helium shell-burning stage begins. The star starts its trip to the Asymptotic Giant Branch (AGB).

Blue Loop Stars

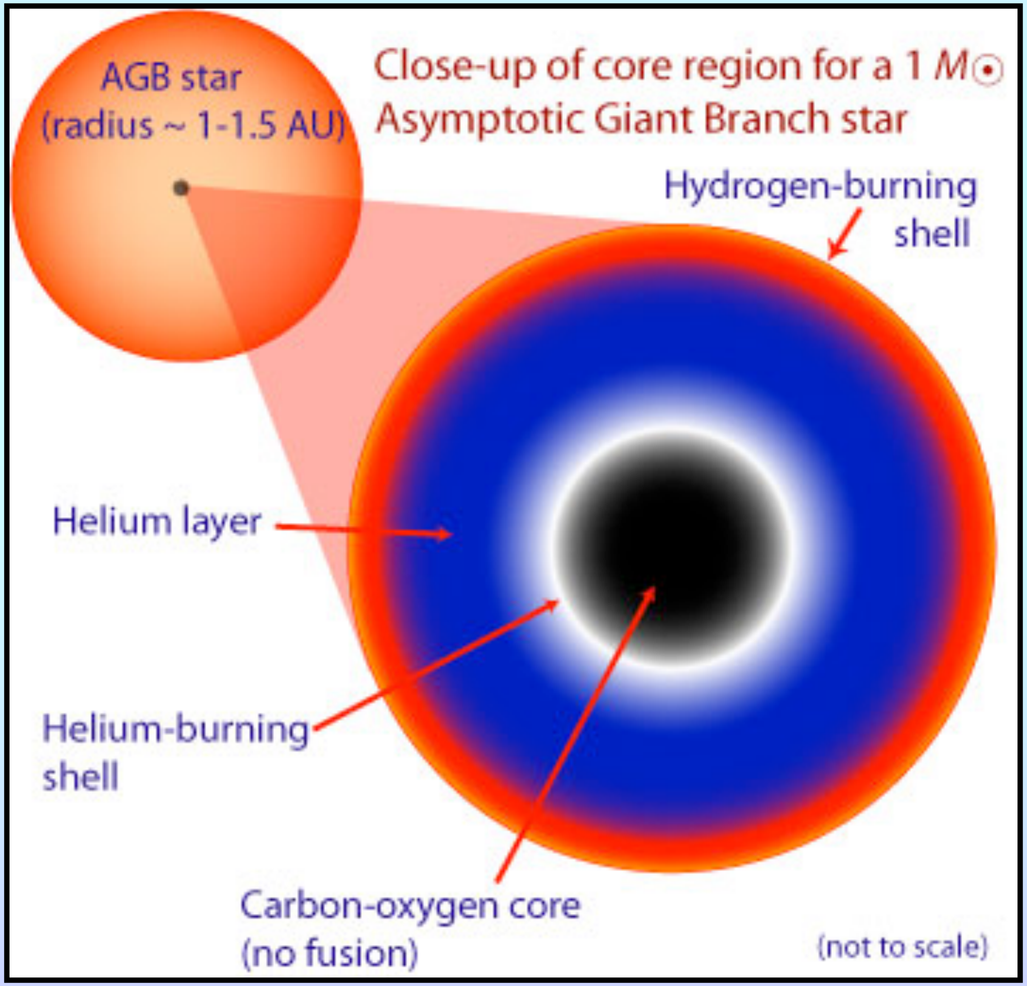
Higher mass stars burn helium in their core non-degenerately; their luminosity is a function of mass, and they evolve through a "blue loop" phase.



Early Asymptotic Giant Branch Stars

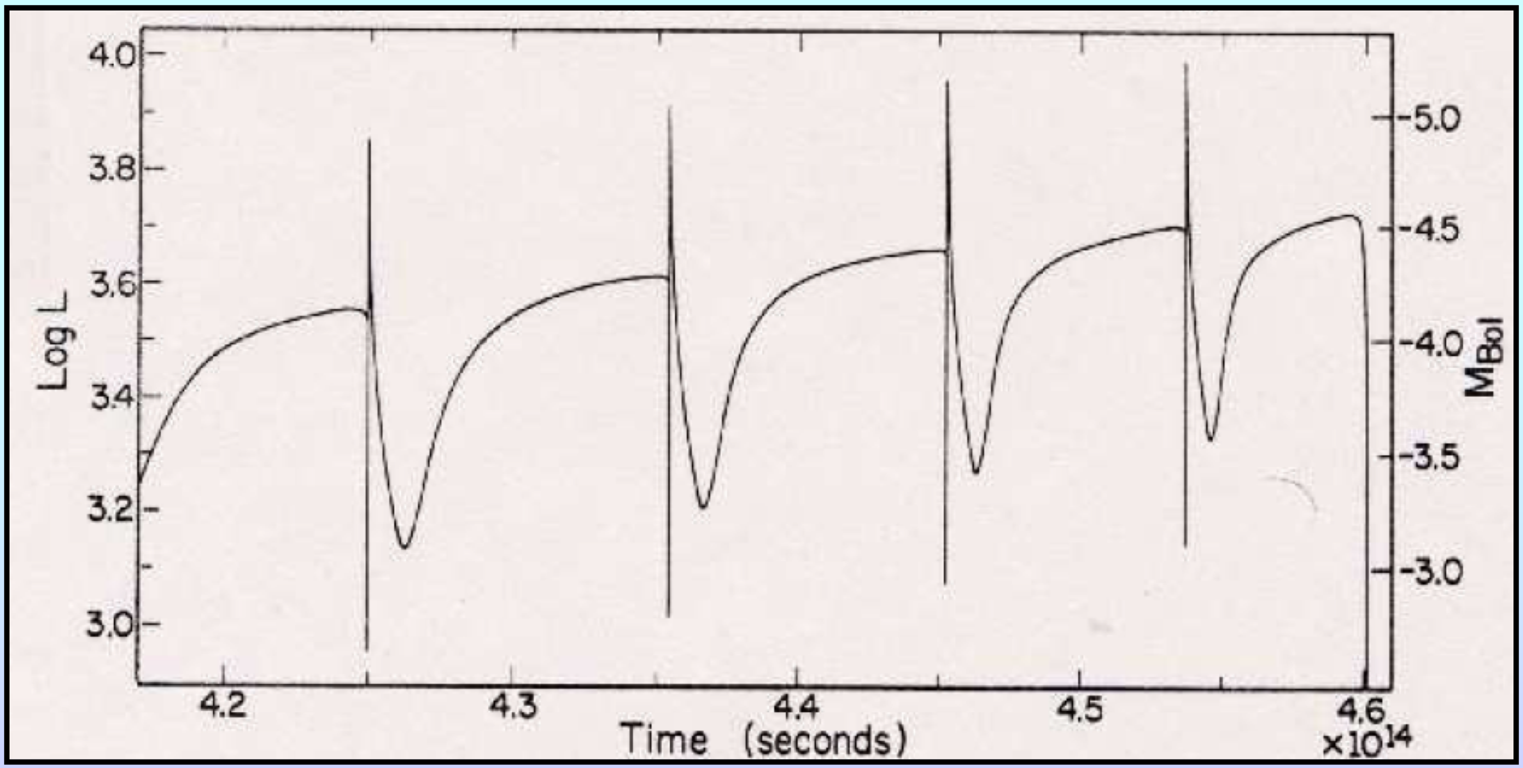
• During its early AGB phase, a star has an inert C/O core, a thick helium burning shell, a hydrogen burning shell, and an envelope that will eventually extend to over 1 A.U.

• The evolution of AGB stars parallels that of RGB stars, with 3 sources of energy: the contracting core, the helium burning shell, and the hydrogen burning shell. The energy for expanding the star comes mostly from the heat of the star itself, through the behavior of Kramers opacity.



Thermal Pulse Asymptotic Giant Branch Stars

- Helium fusion in a thin shell is extremely unstable. When helium ignites in a thin shell, the energy expands the region around it and extinguishes the hydrogen burning shell. After a while, helium shell burning ceases, and the hydrogen shell-burning starts again. These "Thermal Pulses" occur on timescales between 10^5 years and 10 years, depending on the mass of the core.



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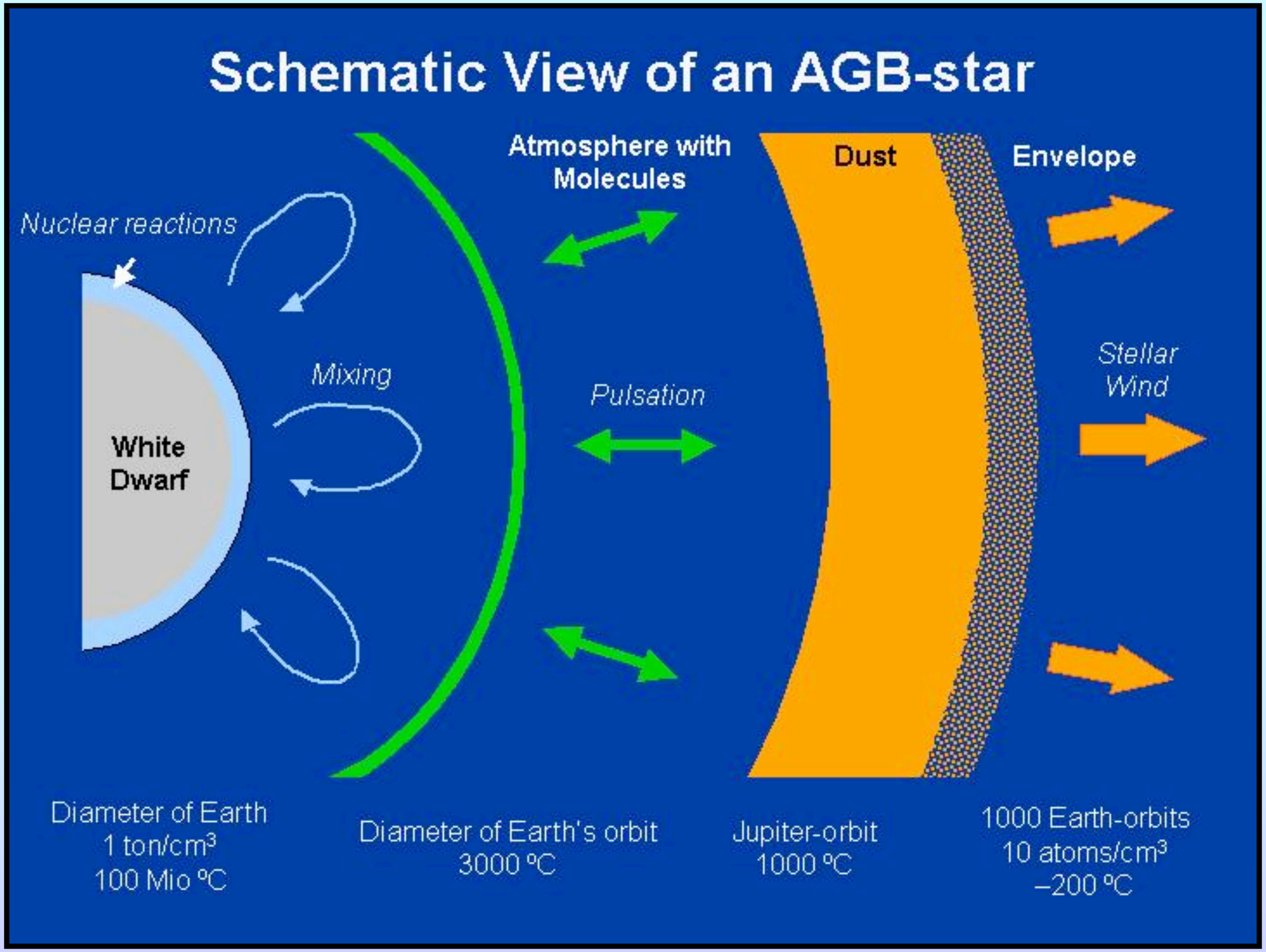
• Because of the thermal pulses, mass loss is greatly enhanced during the TP-AGB phase, and may reach ~ 10^{-4} $M_\odot$/yr. This is called a "superwind," which comes off the star at ~ 10 to 20 km/s.

• In between the pulses, the maximum luminosity of an AGB is proportional to its core mass, with $L \sim 60{,}000\,(M_{core} - 0.52 M_\odot)$.

• Because of the mixing produced by thermal pulses, processed material may be dredged-up to the surface, and CNO processed material may be mixed into the helium-burning shell. Two results will be $^{14}N + {}^4He \rightarrow {}^{18}O + {}^4He \rightarrow {}^{22}Ne + {}^4He \rightarrow {}^{25}Mg + n$, and $^{13}C + {}^4He \rightarrow {}^{16}O + n$. These will be important down the road.

Thermal Pulse Asymptotic Giant Branch Stars

• During the AGB phase, large amounts of dust are created in the star's atmosphere. This dust gets ejected into space during the TP-AGB phase and can enshroud the star (at least in the optical). As a result, TP-AGB stars can be extremely bright in the near and mid-IR.



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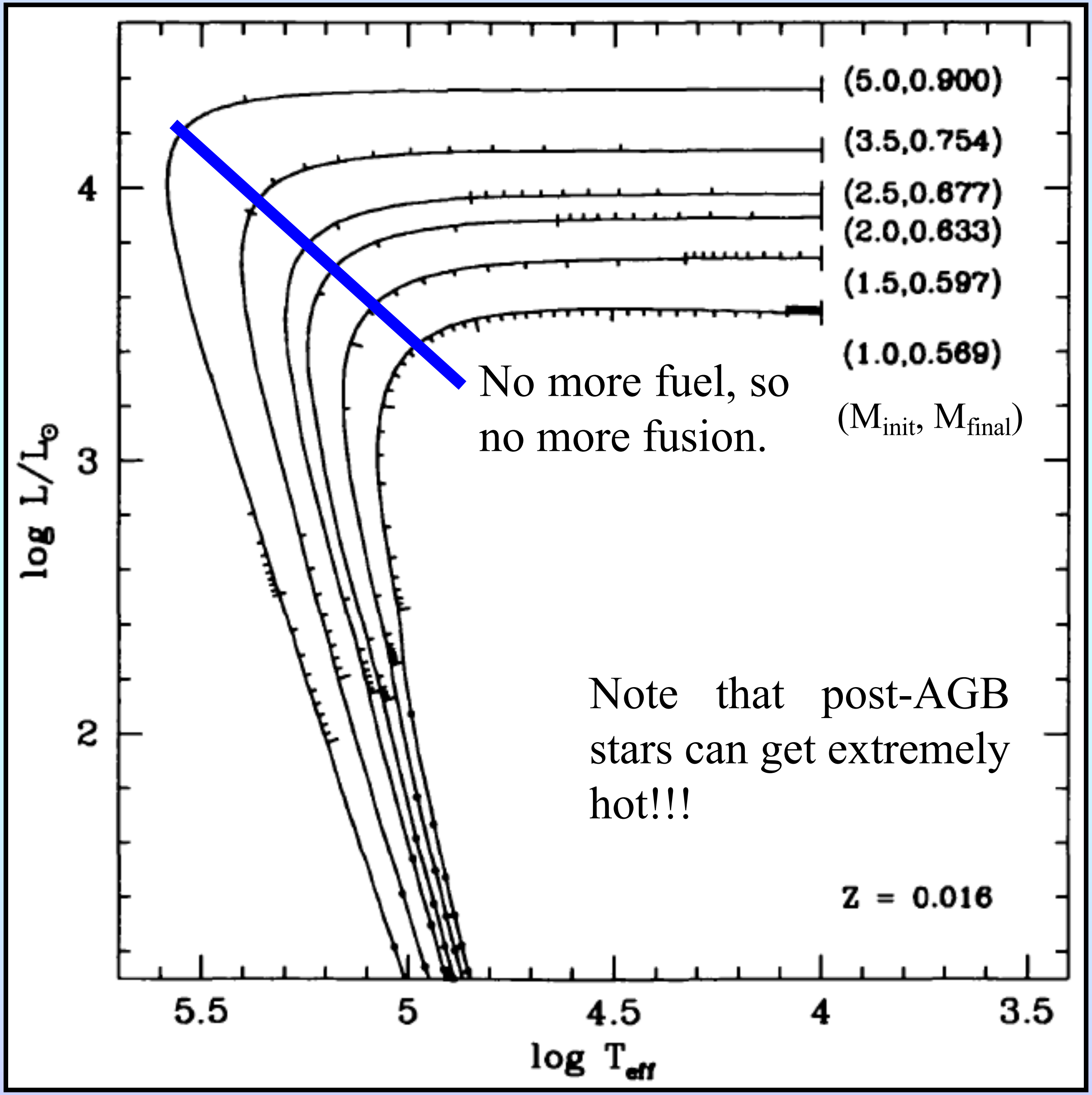
• Because of dredge-up, the relative abundances of C and O can vary greatly from star to star. If O>C, all the C will get tied up in CO; if C>O, various molecules like CH_4 will be observable. (These are called "Carbon stars"). These stars will have very different spectra.

• An AGB's envelope loses mass both due to the H-shell burning (which deposits mass onto the He-core) and mass loss from the star's surface. For lower-mass stars (with initial masses $\lesssim 5\ M_\odot$), the envelope mass will run out before the star's core reaches $1.4\ M_\odot$.

• Horizontal branch stars with extremely small envelope masses may run out of envelope mass before reaching the AGB. These are AGB-manqué stars. Alternatively, they may run out before they start thermal pulsing. These objects become post-EAGB stars.

Post-AGB Evolution

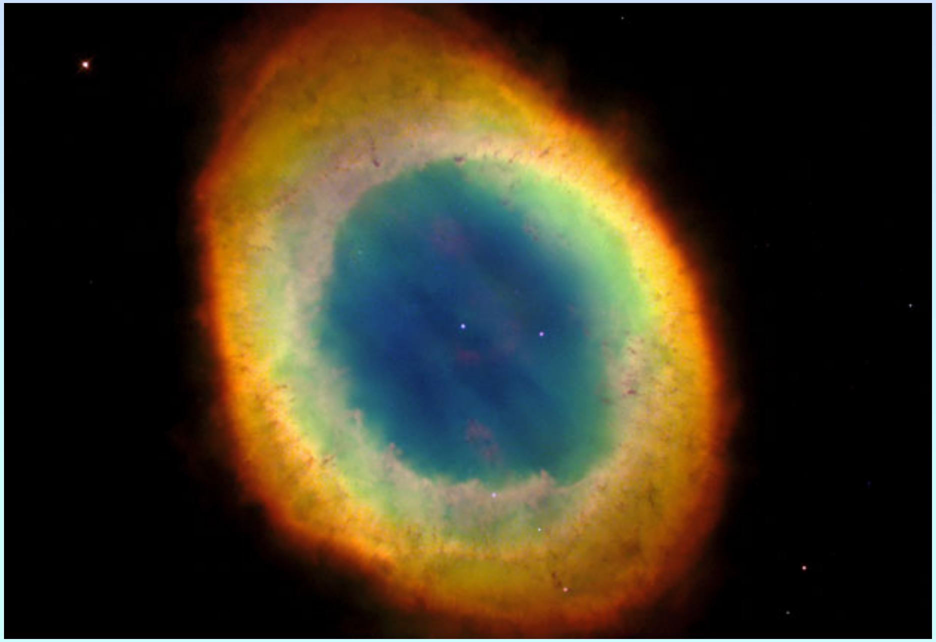
Stars with $M_{\rm init} \lesssim 5M_\odot$ will end up with core masses < 1.4 $M_\odot$. In these objects, C/O will never fuse. But, as the star's envelope mass decreases, so does its optical depth, and as the envelope runs out, the star appears hotter (i.e., moves to the blue in the HR diagram). The rate of post-AGB evolution (and the luminosity at which it occurs) is a strong function of core mass.

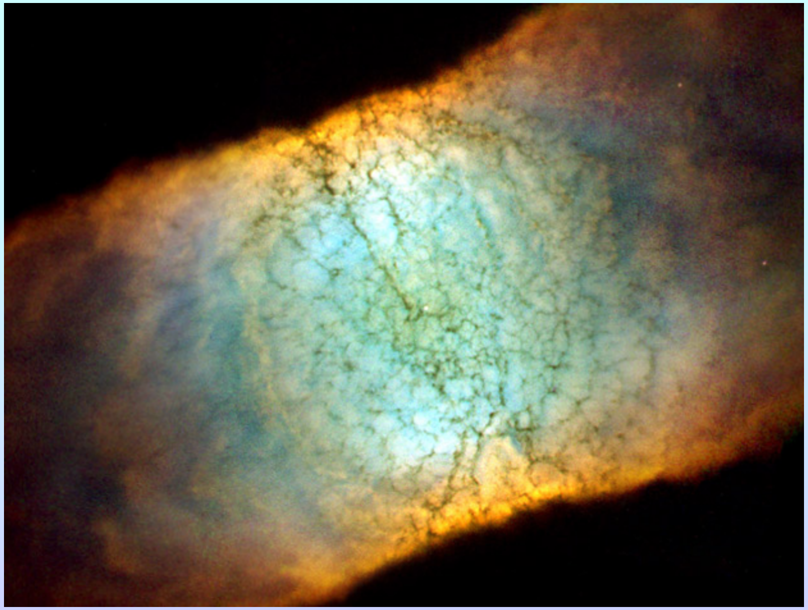


Planetary Nebulae

- As the last of the hydrogen envelope is burned and/or lost to space, the mass loss changes from a slow (~15 km/s) high-density (~10^{-5} $M_\odot$/yr) "superwind" to a fast (~1,000 km/s) low-density (~10^{-11} $M_\odot$/yr) wind. The hot wind blows bubbles in the previous ejecta.
- The timescales of most post-AGB stars are such that the mass lost will still be nearby when the star becomes hot. The high energy photons from the star will ionize the surrounding material. This is called a planetary nebulae (PNe).
- PNe are often asymmetrical, and there is no good theory to explain their shapes other than to invoke some source of angular momentum. Thus, a few/many/most astronomers believe some/many/most/all PNe are produced by interacting binary systems.
- The central stars of planetary nebulae are at the end of their lives. Once fusion stops, they will continue to cool and crystalize for a Hubble time, becoming white dwarf stars.

Planetary Nebulae





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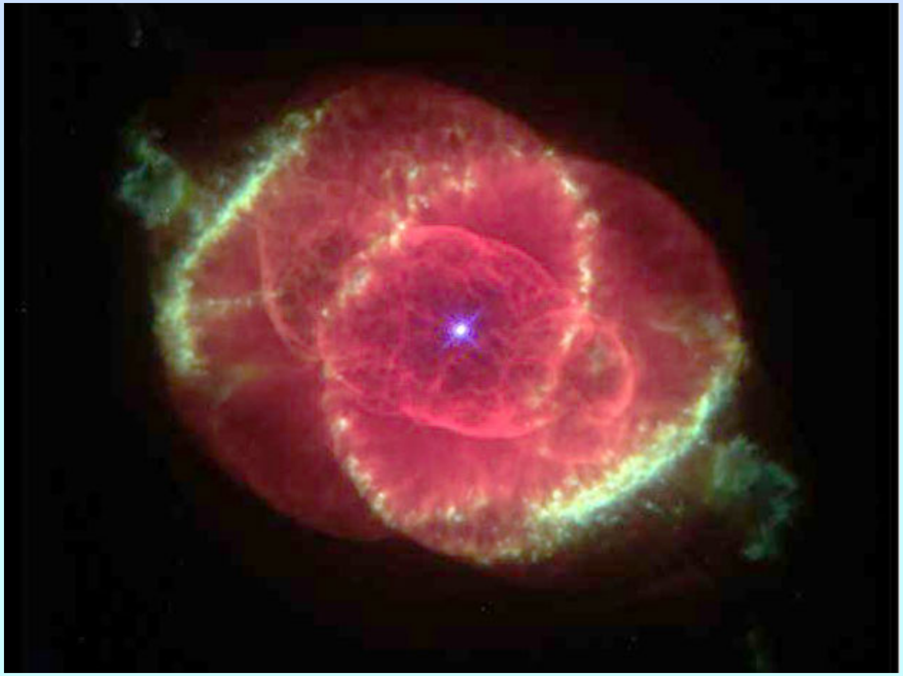
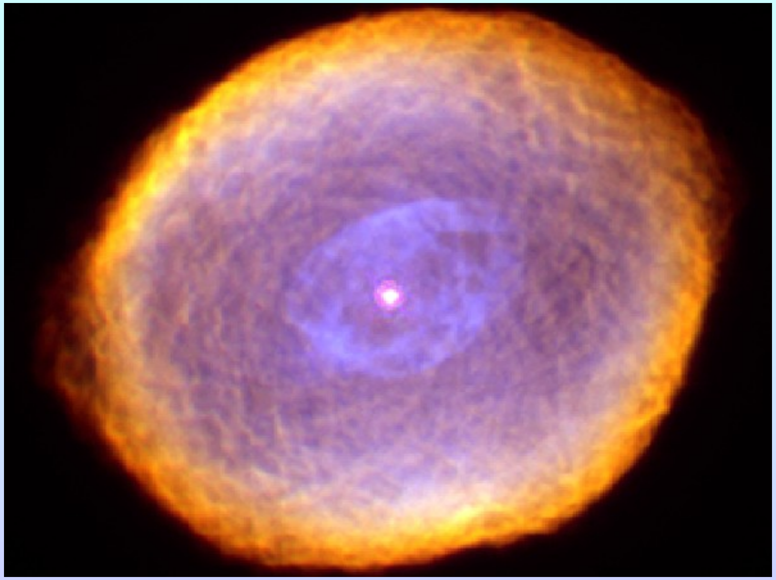
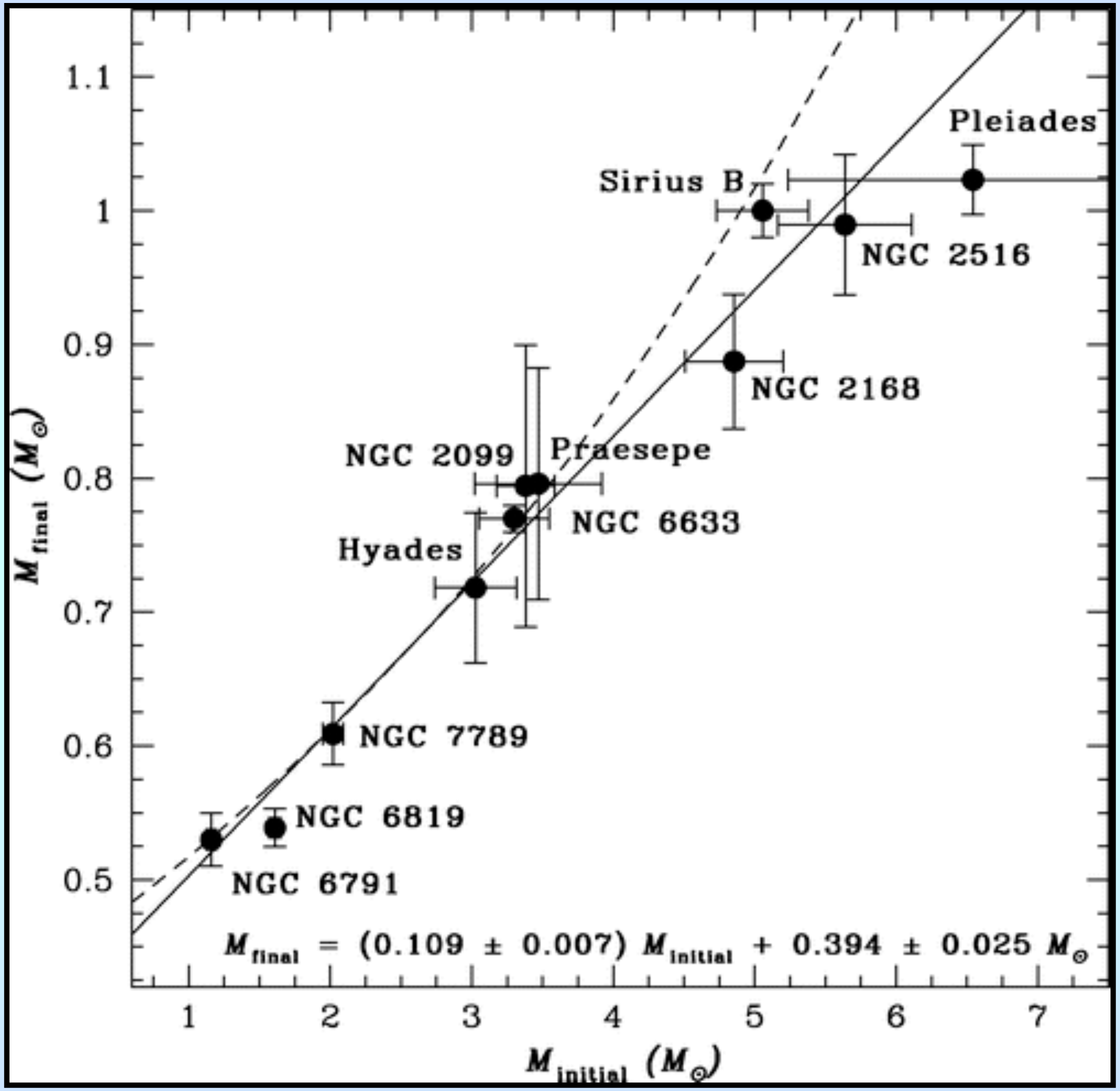




White Dwarfs

Stars with initial masses $M_{\rm init} \lesssim 5\ M_\odot$ end their lives as C/O (and possibly Ne) white dwarfs with masses less than 1.4 $M_\odot$. This is the Chandrasekhar limit.

Note that most white dwarfs are significantly less massive than this. There appears to be a relation between the main sequence mass of a star and its final white dwarf mass.



The Carbon Flash

Electron degeneracy cannot support cores more massive than 1.4 $M_{\odot}$. If a degenerate C/O core (i.e., a white dwarf) is pushed above this limit, it will collapse and begin fusing, but it will not have time to adjust its structure (all the energy will just go into lifting the degeneracy). The star will blow up, converting virtually all of its C/O to iron (and iron-peak) elements. This is likely a Type Ia supernova.

High Mass Stars

High mass stars can fuse carbon/oxygen under non-degenerate conditions. In these objects

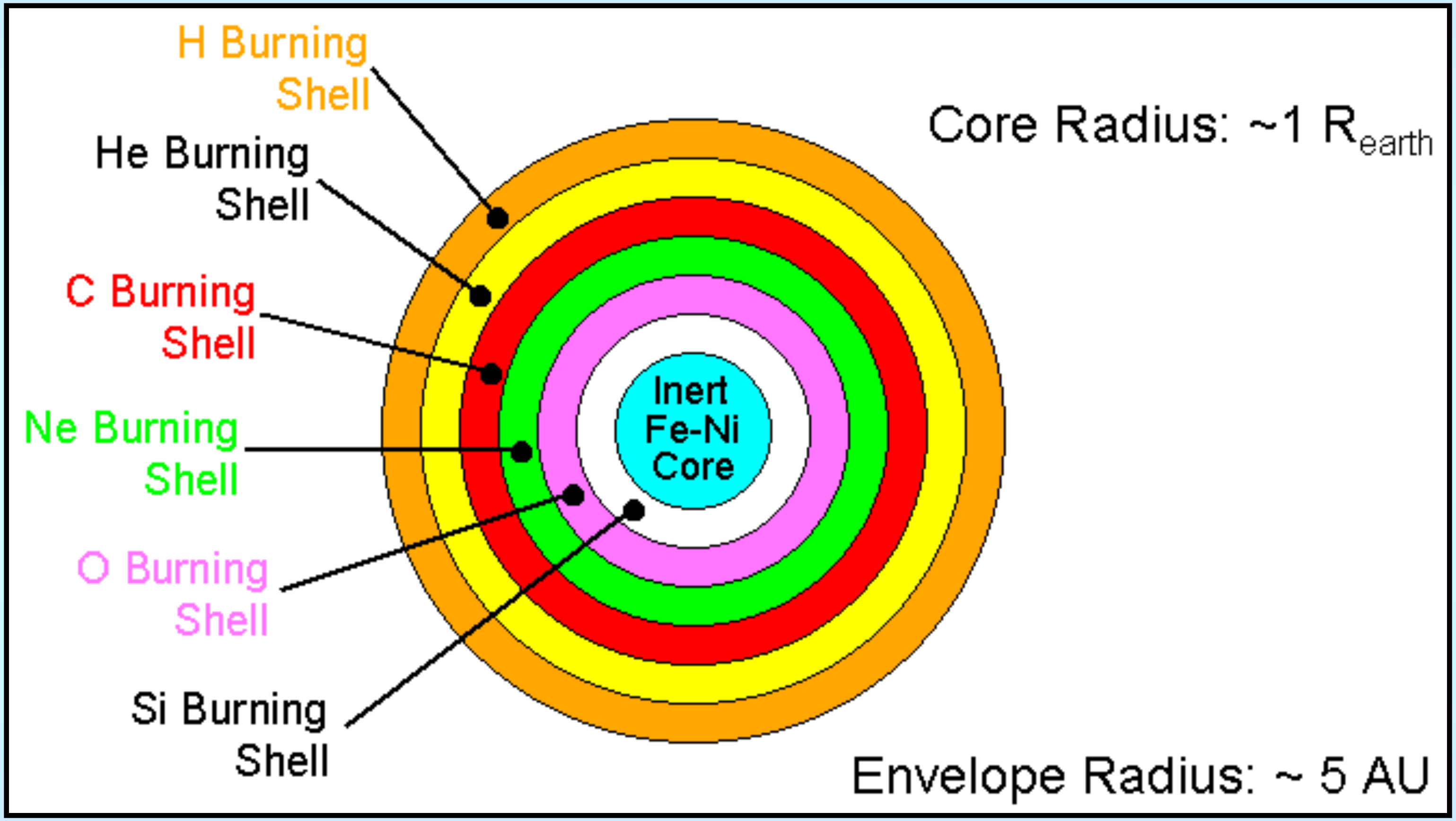
1) Carbon fuses to a number of products, such as Ne, Na, and Mg. This phase lasts for a few years.

2) Oxygen fuses to a number of products, such as S, Ph, and Si. This lasts for about a year.

3) Photodisintegration becomes important. The core temperatures become so high that photons become capable of "ionizing" protons, neutrons, and alpha-particles from the atomic nuclei. Equilibrium amongst the elements is reached, where $X + Y \leftrightarrows Z$ occurs, with the most tightly bound nuclei becoming more abundant. This phase is called "Silicon burning", and it lasts for about a day. The principle product of these fusions (the most tightly bound nucleus) is iron. Note: because iron is the most tightly bound nucleus, any reaction involving iron is endothermic. Once iron is produced, there is no more energy available in the star.

High Mass Stars

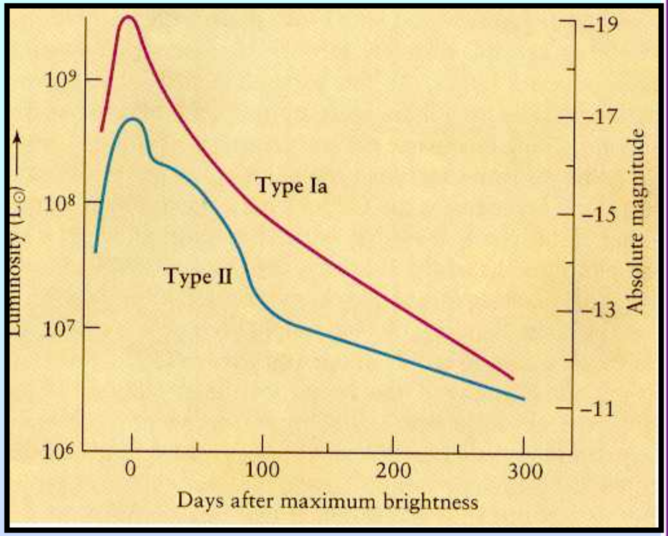
Since the reactions depend on temperature and density, the structure of these high-mass stars resemble an onion-skin.

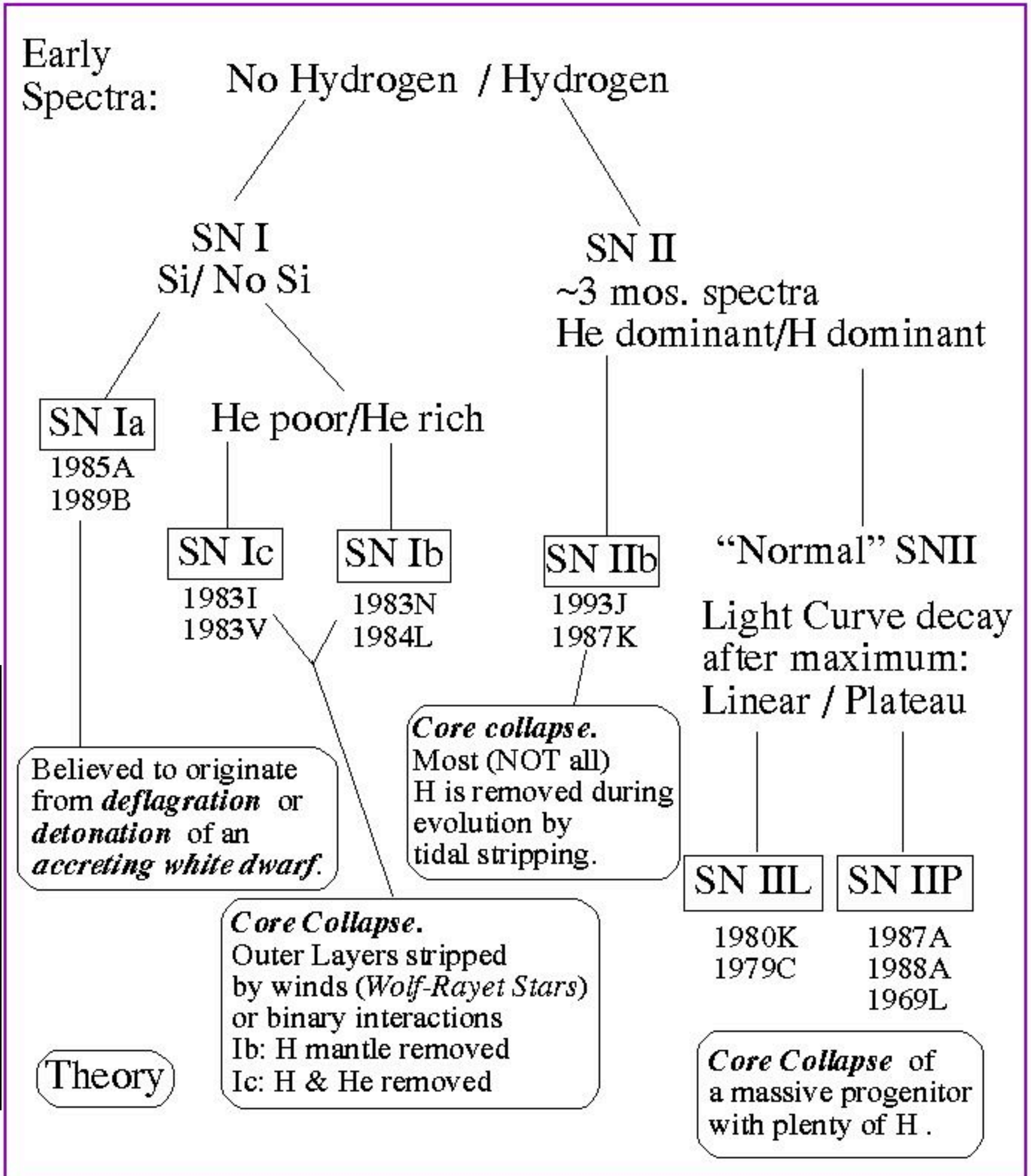


Once iron is made, there is nothing left to support the star. The star collapses on a dynamical timescale, and a core-bounce creates a supernova.

Types of Supernovae

Supernovae types are defined by which elements are seen in the spectra (and by the shape of their light curve).



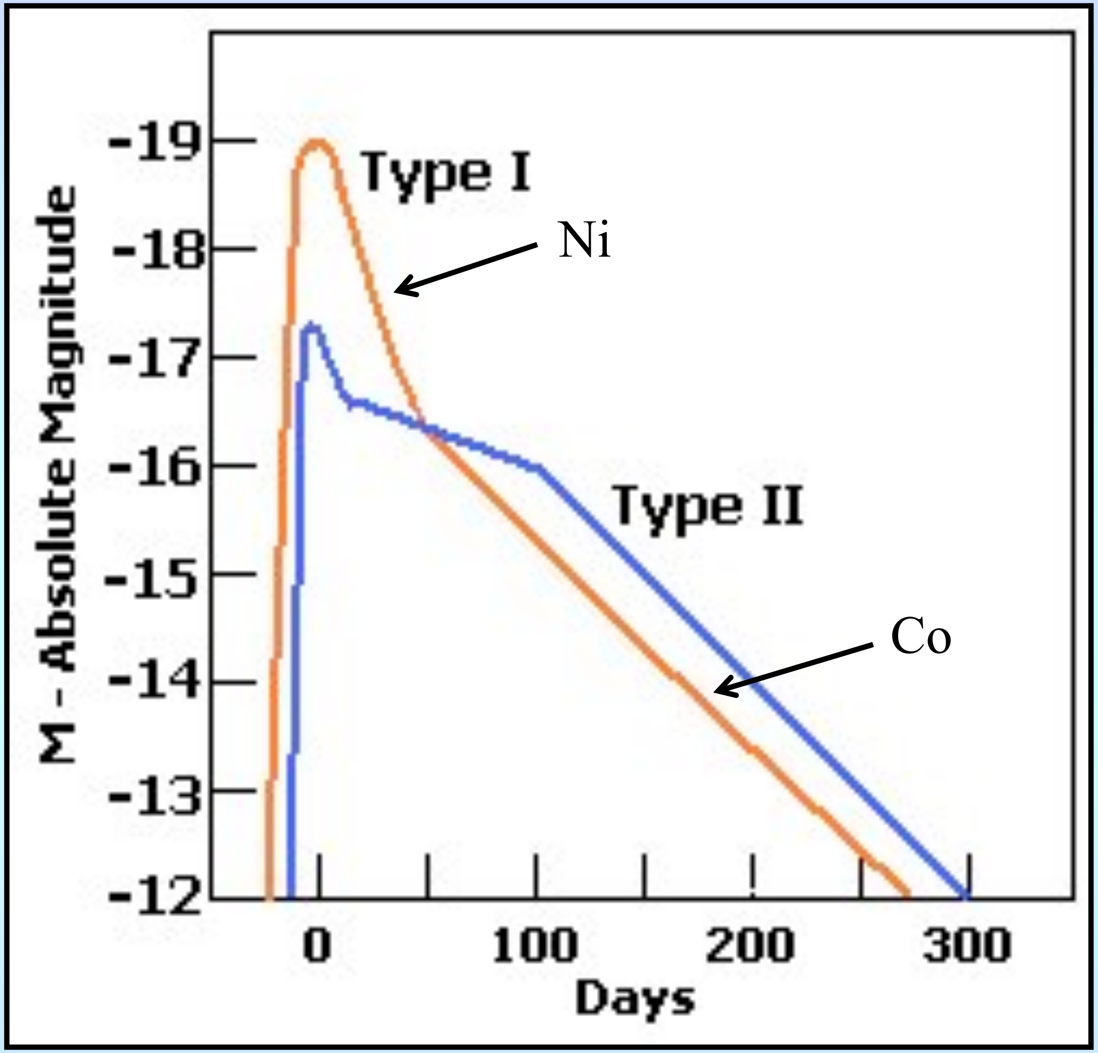


Supernovae After Maximum

After maximum light, a supernova's light curve is powered by the radioactive decay of Ni to Fe.

(Of course, that's the total energy – the amount of light in any particular bandpass may differ.)

Note: the products of SN Ia are almost entirely iron-peak elements (Fe, Ni, etc.) Core-collapse supernovae produce large amounts of light (α-process) elements, such as oxygen.



$$^{56}_{28}\mathrm{Ni} \rightarrow {}^{56}_{27}\mathrm{Co} + e^+ + \nu_e + \gamma \quad (\tau = 6.1 \text{ days})$$

$$^{56}_{27}\mathrm{Co} \rightarrow {}^{56}_{26}\mathrm{Fe} + e^+ + \nu_e + \gamma \quad (\tau = 77.12 \text{ days})$$

Supernovae Luminosities

Note: at maximum, a supernova's luminosity is ~ $10^9\ L_\odot$, and the total amount of light released is ~ 10^{49} ergs. However, this is only ~ 1% of the supernova's total energy – the rest comes out in neutrinos. Many of these neutrinos come from Urca process reactions

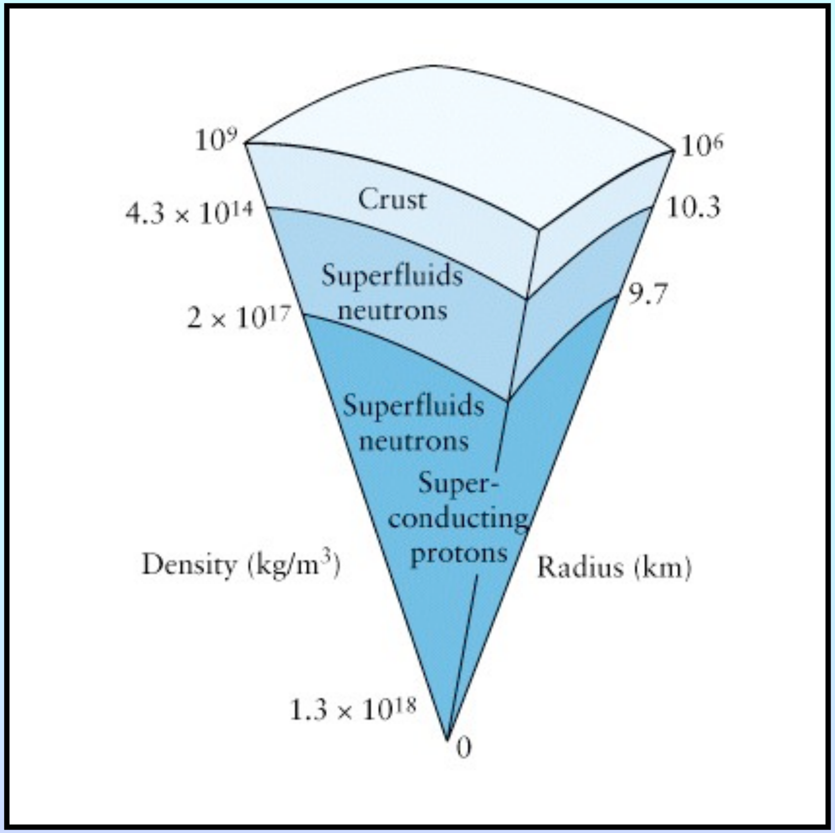
$$ {}^{A}_{Z}X + e^- \rightarrow {}^{A}_{Z-1}Y + \nu_e \qquad {}^{A}_{Z-1}Y \rightarrow {}^{A}_{Z}X + e^- + \bar{\nu}_e $$

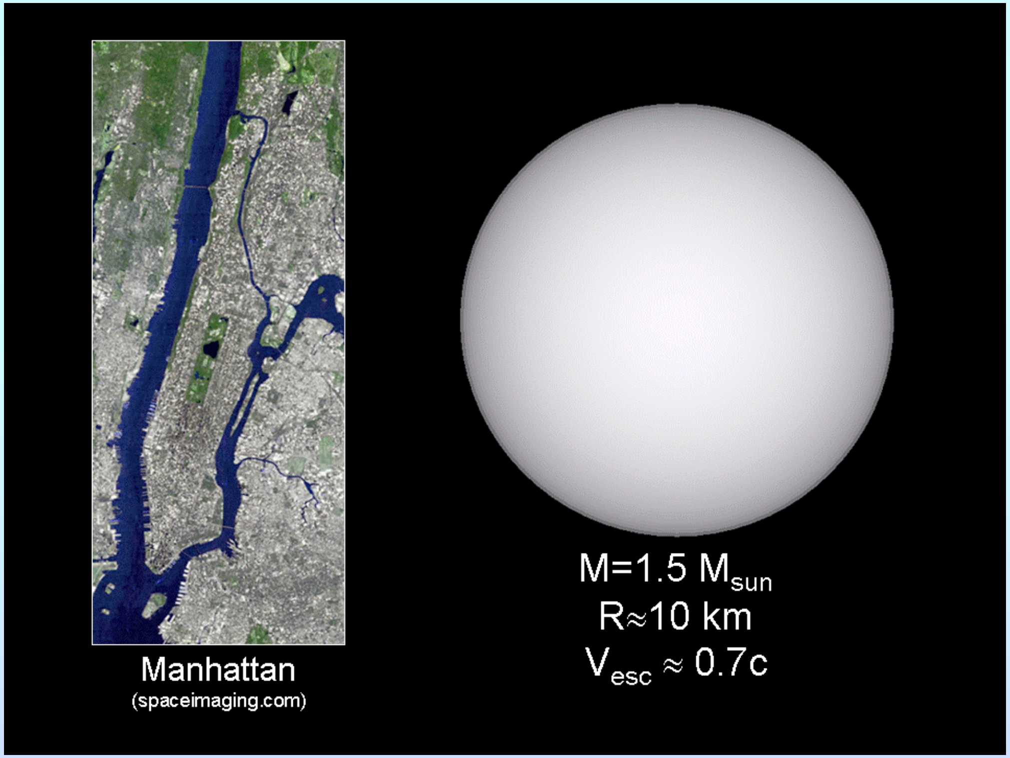
(Note that Urca is not an acronym – it is the name of a casino in Rio de Janiero.)

The Urca process is also responsible for cooling white dwarfs and neutron stars.

Supernova Remnants

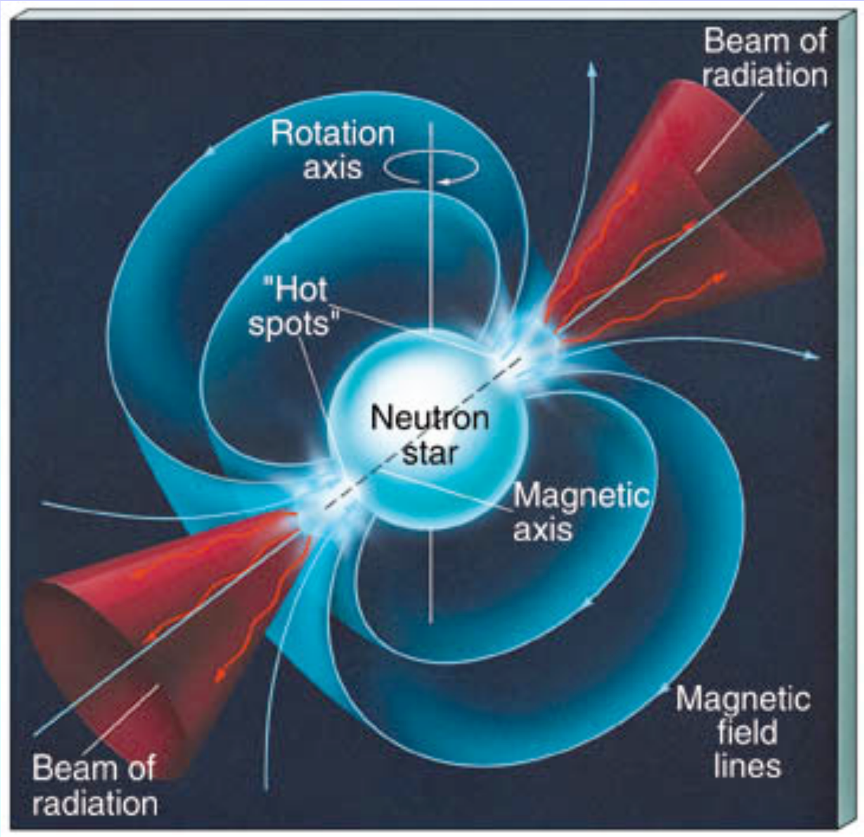
Models of core-collapse supernovae suggest that a massive remnant is produced. If the remnant is less than ~ 3 $M_{\odot}$, then, in theory, it could be supported by neutron degeneracy (i.e., a "neutron star".) Such an star would be only a few miles across, and could be highly magnetized.

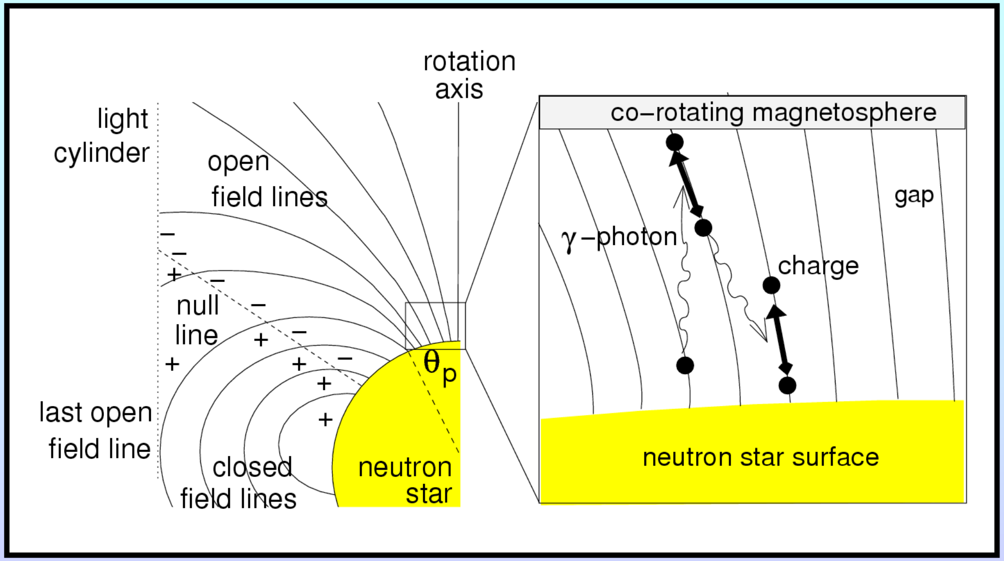




Pulsars

The Sun rotates once every ~ 30 days. If such an object were contracted to a radius of ~ 10 km, then by conservation of angular momentum, $m_1 r^2_1 \Omega_1 = m_2 r^2_2 \Omega_2$, the period will be $P \sim 10^{-4}$ sec.

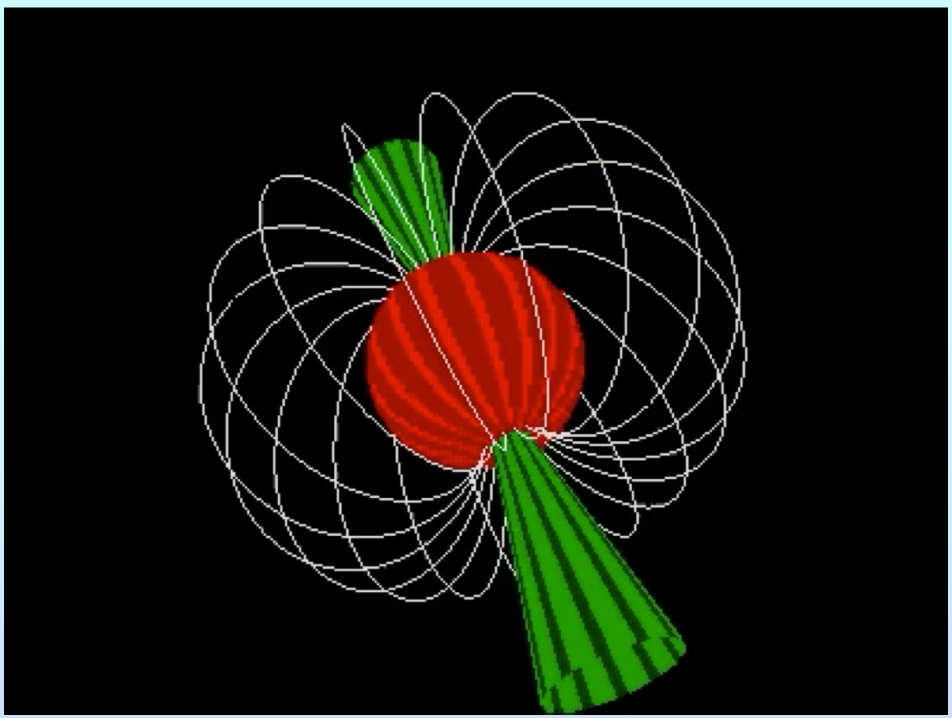


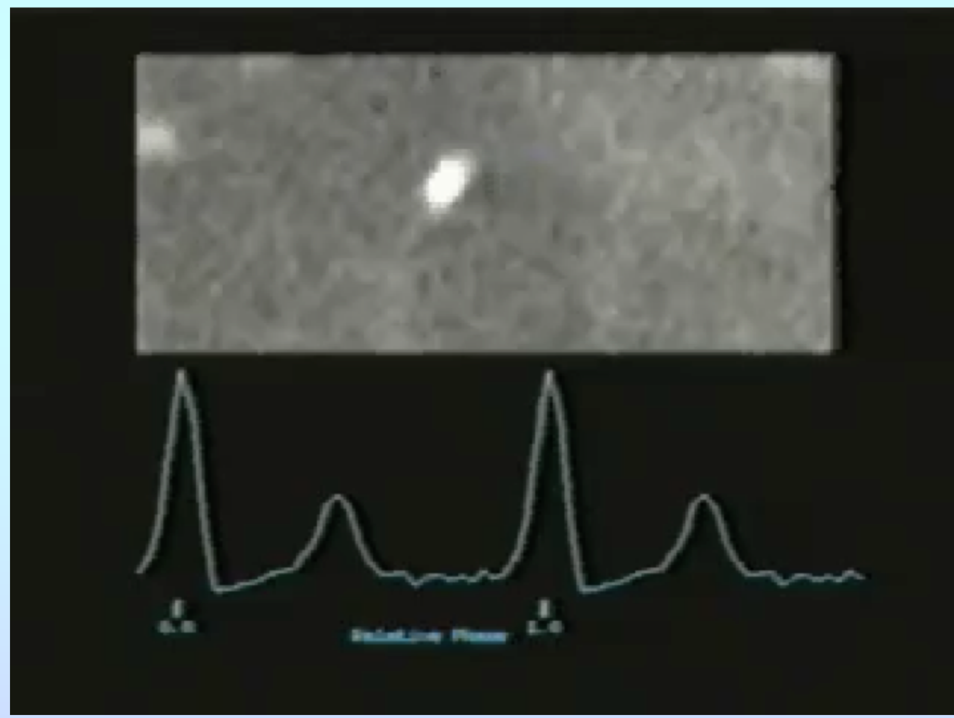


The Sun's magnetic field is roughly 100 G. If such an object were contracted to a radius of ~ 10 km, then by conservation of magnetic flux, $B_1 r^2_1 = B_2 r^2_2$, the B field will be 10^{11} G.

Pulsar Emission

Charged particles trapped in the rotating magnetic field emit via synchrotron emission. Photons escape out the magnetic poles of the object, causing "pulses". This emission can be extremely luminous, ~ 10^{38} ergs/s (though much of this energy comes out at low frequency which is absorbed by the surrounding plasma).





Pulsar Spin-Down

Energy conservation says that the pulsar's radiation must be slowing it down. From Larmor's equation, charged particles radiate in proportion to their acceleration, squared. In the case of a magnetized rotating sphere with angular frequency $\Omega=2\pi/P$ and magnetic field strength B, the magnetic dipole is $m = B R^3$, and the radiated power is

$$p_{\text{rad}} = \frac{2}{3}\frac{q^2 \dot{v}^2}{c^3} = \frac{2}{3}\frac{\left(\ddot{m}_{\perp}\right)^2}{c^3} = \frac{2}{3c^3}\left(\Omega^2 m\right)^2 = \frac{2}{3c^3}\left(BR^3 \sin\alpha\right)^2\left(\frac{2\pi}{P}\right)^4$$

where α is angle between the rotation axis and the magnetic pole. Rotational energy will therefore be lost at a rate of

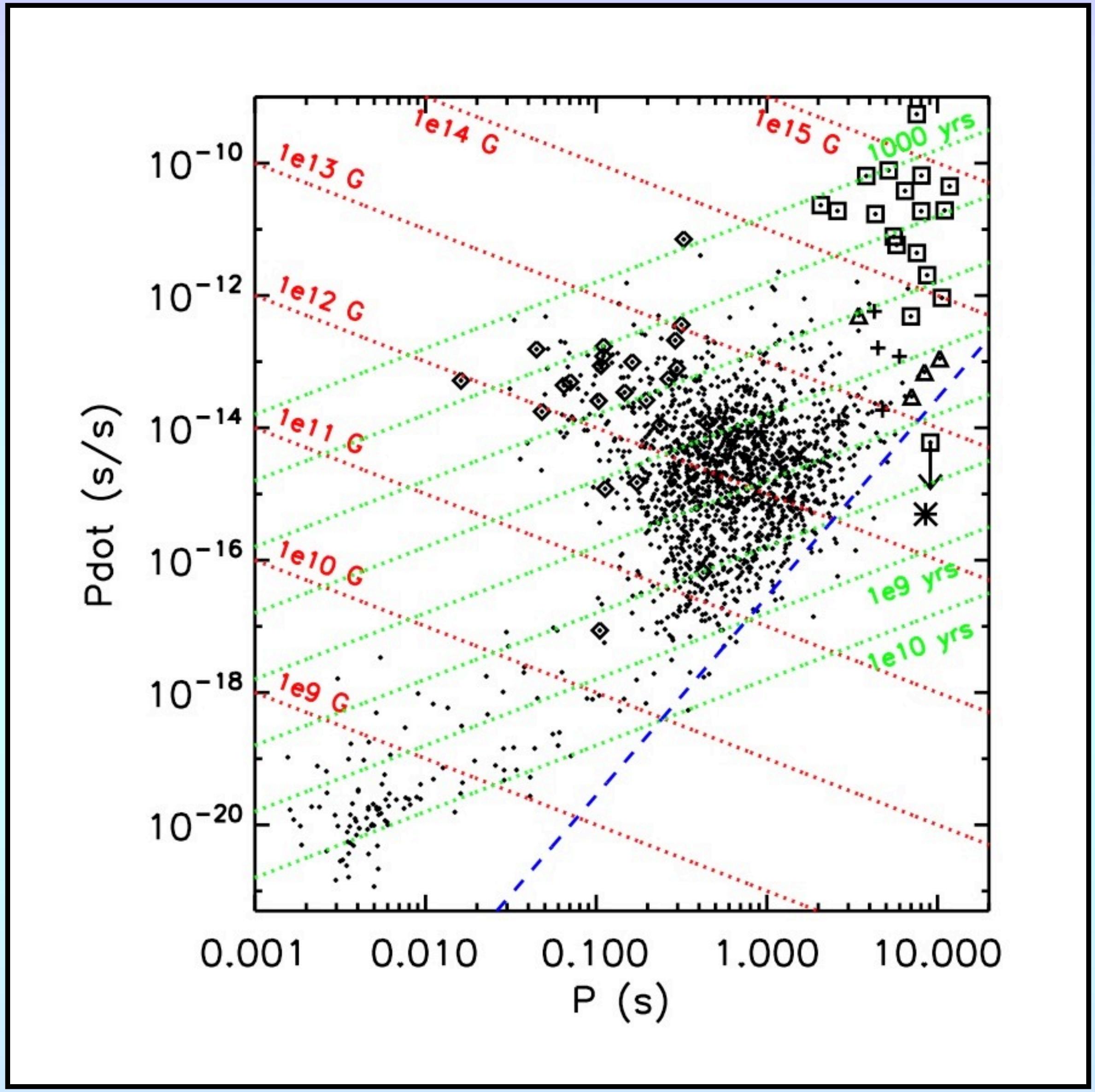
$$E_{rot} = \frac{1}{2} I \Omega^2 \quad \Rightarrow \quad \frac{dE_{rot}}{dt} = I\Omega\dot{\Omega} = -4\pi^2 I \frac{\dot{P}}{P^3} = p_{\text{rad}}$$

where $I \propto MR^2$ is the pulsar's moment of inertia. Consequently,

$$B\sin\alpha = \left(\frac{3c^3 I}{8\pi^2 R^6}\right)^{1/2}\left(P\dot{P}\right)^{1/2}$$

Pulsar Spin-Down

If the pulsar's radius, magnetic field, and moment of inertia do not change with time, and if the initial period P_0 was much smaller than the present-day observed period, P, then the *characteristic age* of the pulsar can be estimated simply from the spin-down rate.



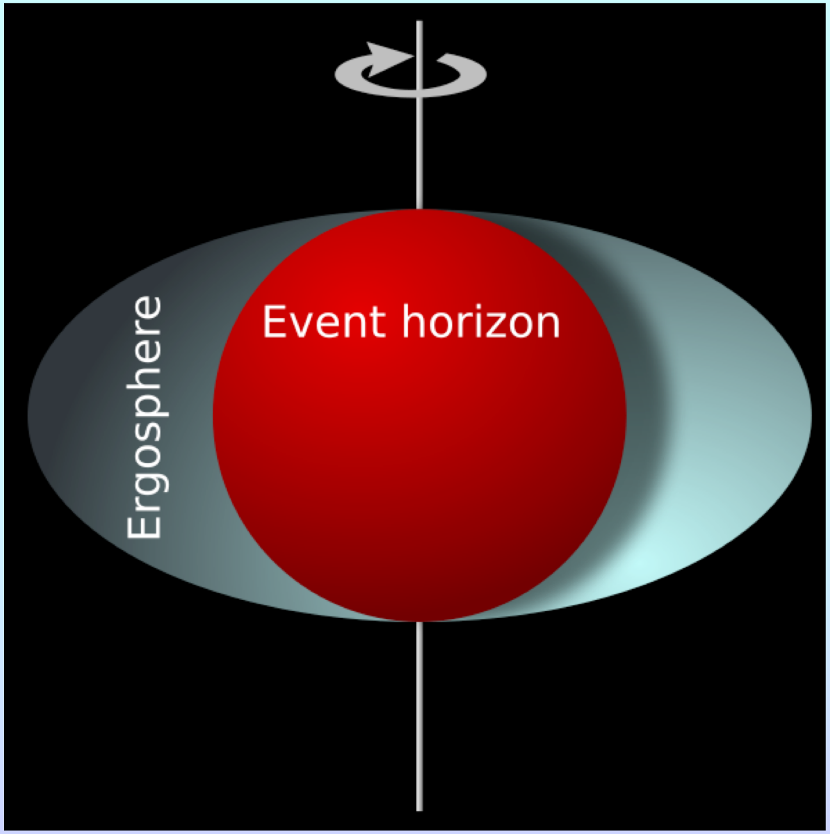
$$P\dot{P} = C \quad \Rightarrow \quad \int_{P_0}^{P'} P\,dP = \int_0^{\tau} C\,dt \quad \Rightarrow \quad \frac{P^2}{2} = C\tau = P\dot{P}\tau \quad \Rightarrow \quad \tau = \frac{P}{2\dot{P}}$$

Black Holes

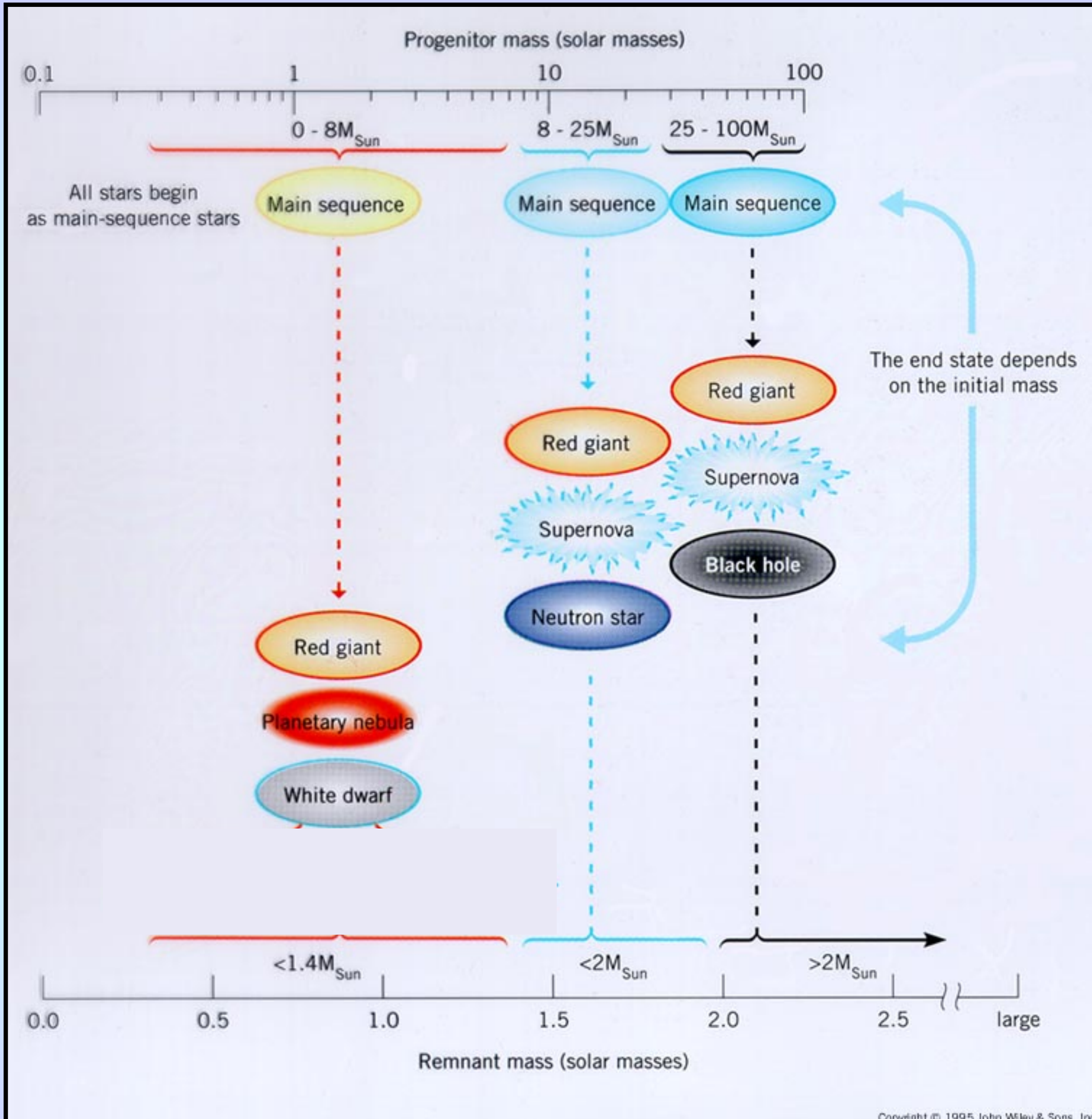
If the mass of a collapsed remnant is greater than ~ 3 $M_\odot$, not even neutron degeneracy can hold it up against gravity. It therefore (theoretically) collapses to a point, i.e., a black hole. The radius where the escape velocity equals the velocity of light is called the Event Horizon or the Schwartzschild radius. For a non-rotating black hole, this is simply

$$\frac{1}{2}mv^2 = \frac{GMm}{R} \quad \Rightarrow \quad R_\bullet = \frac{2GM}{c^2}$$

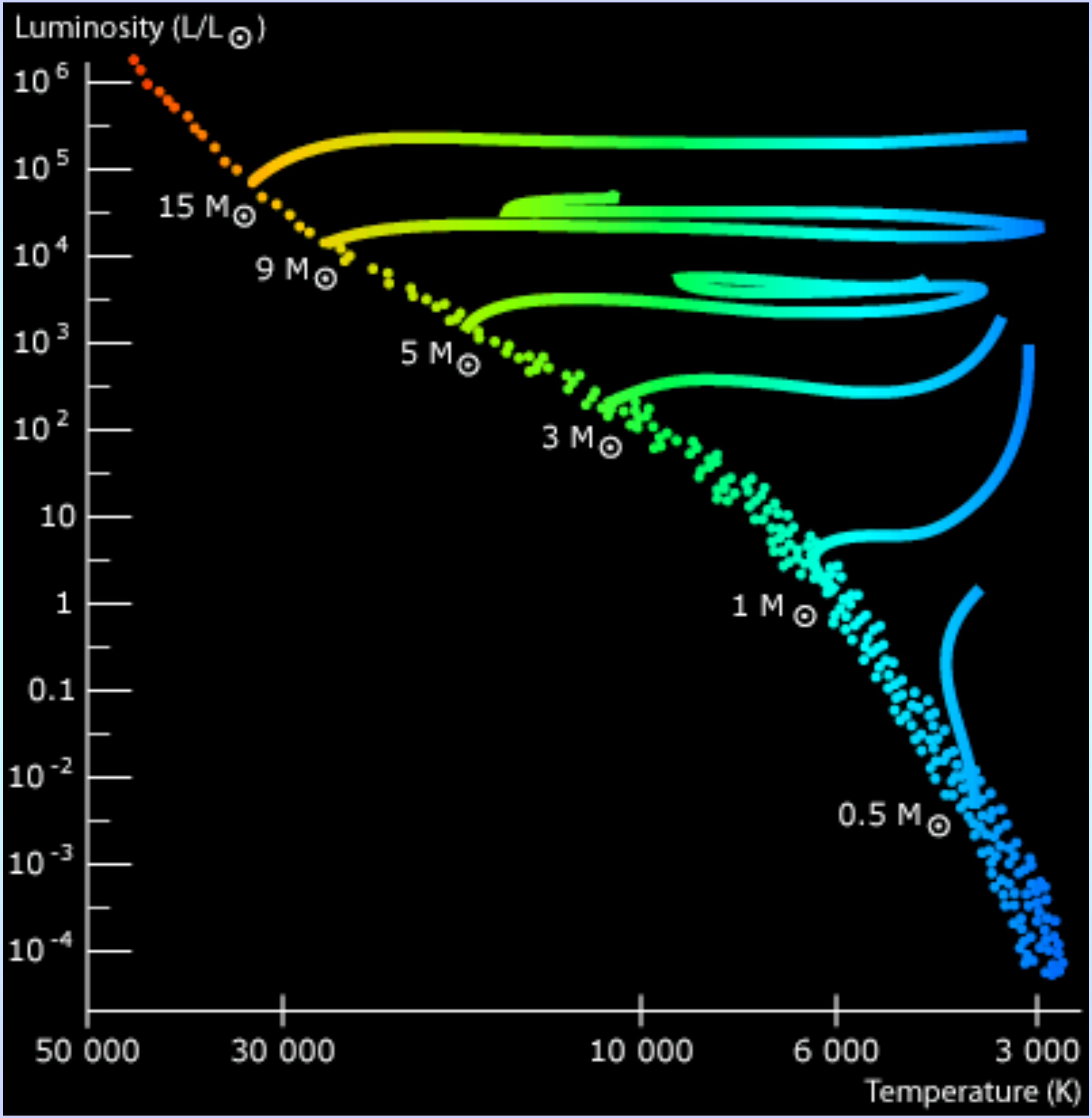
For rotating "Kerr" black holes, the math is a lot more complicated, involving frame dragging and ring-shaped singularities.



Summary of Single-Star Stellar Evolution



Summary of Single-Star Stellar Evolution



PERIODIC TABLE
Atomic Properties of the Elements

NIST National Institute of Standards and Technology, U.S. Department of Commerce

Physical Measurement Laboratory www.nist.gov/pml

Standard Reference Data www.nist.gov/srd

Frequently used fundamental physical constants

For the most accurate values of these and other constants, visit physics.nist.gov/constants

1 second = 9 192 631 770 periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of ^{133}Cs

speed of light in vacuum	c	299 792 458 $\mathrm{m\,s^{-1}}$	(exact)
Planck constant	h	6.626 07 x 10^{-34} J s	($\hbar = h/2\pi$)
elementary charge	e	1.602 177 x 10^{-19} C	
electron mass	m_e	9.109 38 x 10^{-31} kg	
	$m_e c^2$	0.510 999 MeV	
proton mass	m_p	1.672 622 x 10^{-27} kg	
fine-structure constant	α	1/137.035 999	
Rydberg constant	R_∞	10 973 731.569 $\mathrm{m^{-1}}$	
	$R_\infty c$	3.289 841 960 x 10^{15} Hz	
	$R_\infty hc$	13.605 69 eV	
Boltzmann constant	k	1.380 6 x 10^{-23} J $\mathrm{K^{-1}}$	

Solids
Liquids
Gases
Artificially Prepared

Key: Atomic Number; Ground-state Level; Symbol; Name; Standard Atomic Weight[†]; Ground-state Configuration; Ionization Energy (eV)

Example: 58 $^1G^\circ_4$ Ce Cerium 140.116 [Xe]$4f5d6s^2$ 5.5386

Atomic Number	Symbol	Name	Ground-state Level	Standard Atomic Weight	Ground-state Configuration	Ionization Energy (eV)	Period	Group
1	H	Hydrogen	$^2S_{1/2}$	1.008*	$1s$	13.5984	1	1 IA
2	He	Helium	$^1S_0$	4.002602	$1s^2$	24.5874	1	18 VIIIA
3	Li	Lithium	$^2S_{1/2}$	6.94*	$1s^2 2s$	5.3917	2	1 IA
4	Be	Beryllium	$^1S_0$	9.0121831	$1s^2 2s^2$	9.3227	2	2 IIA
5	B	Boron	$^2P^\circ_{1/2}$	10.81*	$1s^2 2s^2 2p$	8.2980	2	13 IIIA
6	C	Carbon	$^3P_0$	12.011*	$1s^2 2s^2 2p^2$	11.2603	2	14 IVA
7	N	Nitrogen	$^4S^\circ_{3/2}$	14.007*	$1s^2 2s^2 2p^3$	14.5341	2	15 VA
8	O	Oxygen	$^3P_2$	15.999*	$1s^2 2s^2 2p^4$	13.6181	2	16 VIA
9	F	Fluorine	$^2P^\circ_{3/2}$	18.99840316	$1s^2 2s^2 2p^5$	17.4228	2	17 VIIA
10	Ne	Neon	$^1S_0$	20.1797	$1s^2 2s^2 2p^6$	21.5645	2	18 VIIIA
11	Na	Sodium	$^2S_{1/2}$	22.98976928	[Ne]$3s$	5.1391	3	1 IA
12	Mg	Magnesium	$^1S_0$	24.305*	[Ne]$3s^2$	7.6462	3	2 IIA
13	Al	Aluminum	$^2P^\circ_{1/2}$	26.9815385	[Ne]$3s^2 3p$	5.9858	3	13 IIIA
14	Si	Silicon	$^3P_0$	28.085*	[Ne]$3s^2 3p^2$	8.1517	3	14 IVA
15	P	Phosphorus	$^4S^\circ_{3/2}$	30.97376200	[Ne]$3s^2 3p^3$	10.4867	3	15 VA
16	S	Sulfur	$^3P_2$	32.06*	[Ne]$3s^2 3p^4$	10.3600	3	16 VIA
17	Cl	Chlorine	$^2P^\circ_{3/2}$	35.45*	[Ne]$3s^2 3p^5$	12.9676	3	17 VIIA
18	Ar	Argon	$^1S_0$	39.948	[Ne]$3s^2 3p^6$	15.7596	3	18 VIIIA
19	K	Potassium	$^2S_{1/2}$	39.0983	[Ar]$4s$	4.3407	4	1 IA
20	Ca	Calcium	$^1S_0$	40.078	[Ar]$4s^2$	6.1132	4	2 IIA
21	Sc	Scandium	$^2D_{3/2}$	44.955908	[Ar]$3d4s^2$	6.5615	4	3 IIIB
22	Ti	Titanium	$^3F_2$	47.867	[Ar]$3d^2 4s^2$	6.8281	4	4 IVB
23	V	Vanadium	$^4F_{3/2}$	50.9415	[Ar]$3d^3 4s^2$	6.7462	4	5 VB
24	Cr	Chromium	$^7S_3$	51.9961	[Ar]$3d^5 4s$	6.7665	4	6 VIB
25	Mn	Manganese	$^6S_{5/2}$	54.938044	[Ar]$3d^5 4s^2$	7.4340	4	7 VIIB
26	Fe	Iron	$^5D_4$	55.845	[Ar]$3d^6 4s^2$	7.9025	4	8 VIII
27	Co	Cobalt	$^4F_{9/2}$	58.933194	[Ar]$3d^7 4s^2$	7.8810	4	9 VIII
28	Ni	Nickel	$^3F_4$	58.6934	[Ar]$3d^8 4s^2$	7.6399	4	10 VIII
29	Cu	Copper	$^2S_{1/2}$	63.546	[Ar]$3d^{10} 4s$	7.7264	4	11 IB
30	Zn	Zinc	$^1S_0$	65.38	[Ar]$3d^{10} 4s^2$	9.3942	4	12 IIB
31	Ga	Gallium	$^2P^\circ_{1/2}$	69.723	[Ar]$3d^{10} 4s^2 4p$	5.9993	4	13 IIIA
32	Ge	Germanium	$^3P_0$	72.630	[Ar]$3d^{10} 4s^2 4p^2$	7.8994	4	14 IVA
33	As	Arsenic	$^4S^\circ_{3/2}$	74.921595	[Ar]$3d^{10} 4s^2 4p^3$	9.7886	4	15 VA
34	Se	Selenium	$^3P_2$	78.971	[Ar]$3d^{10} 4s^2 4p^4$	9.7524	4	16 VIA
35	Br	Bromine	$^2P^\circ_{3/2}$	79.904*	[Ar]$3d^{10} 4s^2 4p^5$	11.8138	4	17 VIIA
36	Kr	Krypton	$^1S_0$	83.798	[Ar]$3d^{10} 4s^2 4p^6$	13.9996	4	18 VIIIA
37	Rb	Rubidium	$^2S_{1/2}$	85.4678	[Kr]$5s$	4.1771	5	1 IA
38	Sr	Strontium	$^1S_0$	87.62	[Kr]$5s^2$	5.6949	5	2 IIA
39	Y	Yttrium	$^2D_{3/2}$	88.90584	[Kr]$4d5s^2$	6.2173	5	3 IIIB
40	Zr	Zirconium	$^3F_2$	91.224	[Kr]$4d^2 5s^2$	6.6339	5	4 IVB
41	Nb	Niobium	$^6D_{1/2}$	92.90637	[Kr]$4d^4 5s$	6.7589	5	5 VB
42	Mo	Molybdenum	$^7S_3$	95.95	[Kr]$4d^5 5s$	7.0924	5	6 VIB
43	Tc	Technetium	$^6S_{5/2}$	(98)	[Kr]$4d^5 5s^2$	7.1194	5	7 VIIB
44	Ru	Ruthenium	$^5F_5$	101.07	[Kr]$4d^7 5s$	7.3605	5	8 VIII
45	Rh	Rhodium	$^4F_{9/2}$	102.90550	[Kr]$4d^8 5s$	7.4589	5	9 VIII
46	Pd	Palladium	$^1S_0$	106.42	[Kr]$4d^{10}$	8.3369	5	10 VIII
47	Ag	Silver	$^2S_{1/2}$	107.8682	[Kr]$4d^{10} 5s$	7.5762	5	11 IB
48	Cd	Cadmium	$^1S_0$	112.414	[Kr]$4d^{10} 5s^2$	8.9938	5	12 IIB
49	In	Indium	$^2P^\circ_{1/2}$	114.818	[Kr]$4d^{10} 5s^2 5p$	5.7864	5	13 IIIA
50	Sn	Tin	$^3P_0$	118.710	[Kr]$4d^{10} 5s^2 5p^2$	7.3439	5	14 IVA
51	Sb	Antimony	$^4S^\circ_{3/2}$	121.760	[Kr]$4d^{10} 5s^2 5p^3$	8.6084	5	15 VA
52	Te	Tellurium	$^3P_2$	127.60	[Kr]$4d^{10} 5s^2 5p^4$	9.0097	5	16 VIA
53	I	Iodine	$^2P^\circ_{3/2}$	126.90447	[Kr]$4d^{10} 5s^2 5p^5$	10.4513	5	17 VIIA
54	Xe	Xenon	$^1S_0$	131.293	[Kr]$4d^{10} 5s^2 5p^6$	12.1298	5	18 VIIIA
55	Cs	Cesium	$^2S_{1/2}$	132.9054520	[Xe]$6s$	3.8939	6	1 IA
56	Ba	Barium	$^1S_0$	137.327	[Xe]$6s^2$	5.2117	6	2 IIA
72	Hf	Hafnium	$^3F_2$	178.49	[Xe]$4f^{14} 5d^2 6s^2$	6.8251	6	4 IVB
73	Ta	Tantalum	$^4F_{3/2}$	180.94788	[Xe]$4f^{14} 5d^3 6s^2$	7.5496	6	5 VB
74	W	Tungsten	$^5D_0$	183.84	[Xe]$4f^{14} 5d^4 6s^2$	7.8640	6	6 VIB
75	Re	Rhenium	$^6S_{5/2}$	186.207	[Xe]$4f^{14} 5d^5 6s^2$	7.8335	6	7 VIIB
76	Os	Osmium	$^5D_4$	190.23	[Xe]$4f^{14} 5d^6 6s^2$	8.4382	6	8 VIII
77	Ir	Iridium	$^4F_{9/2}$	192.217	[Xe]$4f^{14} 5d^7 6s^2$	8.9670	6	9 VIII
78	Pt	Platinum	$^3D_3$	195.084	[Xe]$4f^{14} 5d^9 6s$	8.9588	6	10 VIII
79	Au	Gold	$^2S_{1/2}$	196.966569	[Xe]$4f^{14} 5d^{10} 6s$	9.2256	6	11 IB
80	Hg	Mercury	$^1S_0$	200.592	[Xe]$4f^{14} 5d^{10} 6s^2$	10.4375	6	12 IIB
81	Tl	Thallium	$^2P^\circ_{1/2}$	204.38*	[Hg]$6p$	6.1083	6	13 IIIA
82	Pb	Lead	$^3P_0$	207.2	[Hg]$6p^2$	7.4167	6	14 IVA
83	Bi	Bismuth	$^4S^\circ_{3/2}$	208.98040	[Hg]$6p^3$	7.2855	6	15 VA
84	Po	Polonium	$^3P_2$	(209)	[Hg]$6p^4$	8.414	6	16 VIA
85	At	Astatine	$^2P^\circ_{3/2}$	(210)	[Hg]$6p^5$	9.31751	6	17 VIIA
86	Rn	Radon	$^1S_0$	(222)	[Hg]$6p^6$	10.7485	6	18 VIIIA
87	Fr	Francium	$^2S_{1/2}$	(223)	[Rn]$7s$	4.0727	7	1 IA
88	Ra	Radium	$^1S_0$	(226)	[Rn]$7s^2$	5.2784	7	2 IIA
104	Rf	Rutherfordium	$^3F_2$	(267)	[Rn]$5f^{14} 6d^2 7s^2$	6.01	7	4 IVB
105	Db	Dubnium	$^4F_{3/2}$	(268)	[Rn]$5f^{14} 6d^3 7s^2$	6.8	7	5 VB
106	Sg	Seaborgium		(271)	[Rn]$5f^{14} 6d^4 7s^2$	7.8	7	6 VIB
107	Bh	Bohrium		(272)	[Rn]$5f^{14} 6d^5 7s^2$	7.7	7	7 VIIB
108	Hs	Hassium		(270)	[Rn]$5f^{14} 6d^6 7s^2$	7.6	7	8 VIII
109	Mt	Meitnerium		(276)			7	9 VIII
110	Ds	Darmstadtium		(281)			7	10 VIII
111	Rg	Roentgenium		(280)			7	11 IB
112	Cn	Copernicium		(285)			7	12 IIB
113	Uut	Ununtrium		(284)			7	13 IIIA
114	Fl	Flerovium		(289)			7	14 IVA
115	Uup	Ununpentium		(288)			7	15 VA
116	Lv	Livermorium		(293)			7	16 VIA
117	Uus	Ununseptium		(294)			7	17 VIIA
118	Uuo	Ununoctium		(294)			7	18 VIIIA
57	La	Lanthanum	$^2D_{3/2}$	138.90547	[Xe]$5d6s^2$	5.5769	Lanthanides	
58	Ce	Cerium	$^1G^\circ_4$	140.116	[Xe]$4f5d6s^2$	5.5386	Lanthanides	
59	Pr	Praseodymium	$^4I^\circ_{9/2}$	140.907	[Xe]$4f^3 6s^2$	5.473	Lanthanides	
60	Nd	Neodymium	$^5I_4$	144.242	[Xe]$4f^4 6s^2$	5.5250	Lanthanides	
61	Pm	Promethium	$^6H^\circ_{5/2}$	(145)	[Xe]$4f^5 6s^2$	5.582	Lanthanides	
62	Sm	Samarium	$^7F_0$	150.36	[Xe]$4f^6 6s^2$	5.6437	Lanthanides	
63	Eu	Europium	$^8S^\circ_{7/2}$	151.964	[Xe]$4f^7 6s^2$	5.6704	Lanthanides	
64	Gd	Gadolinium	$^9D^\circ_2$	157.25	[Xe]$4f^7 5d6s^2$	6.1498	Lanthanides	
65	Tb	Terbium	$^6H^\circ_{15/2}$	158.92535	[Xe]$4f^9 6s^2$	5.8638	Lanthanides	
66	Dy	Dysprosium	$^5I_8$	162.500	[Xe]$4f^{10} 6s^2$	5.9391	Lanthanides	
67	Ho	Holmium	$^4I^\circ_{15/2}$	164.93033	[Xe]$4f^{11} 6s^2$	6.0215	Lanthanides	
68	Er	Erbium	$^3H_6$	167.259	[Xe]$4f^{12} 6s^2$	6.1077	Lanthanides	
69	Tm	Thulium	$^2F^\circ_{7/2}$	168.93422	[Xe]$4f^{13} 6s^2$	6.1843	Lanthanides	
70	Yb	Ytterbium	$^1S_0$	173.054	[Xe]$4f^{14} 6s^2$	6.2542	Lanthanides	
71	Lu	Lutetium	$^2D_{3/2}$	174.9668	[Xe]$4f^{14} 5d6s^2$	5.4259	Lanthanides	
89	Ac	Actinium	$^2D_{3/2}$	(227)	[Rn]$6d7s^2$	5.3802	Actinides	
90	Th	Thorium	$^3F_2$	232.0377	[Rn]$6d^2 7s^2$	6.3067	Actinides	
91	Pa	Protactinium	$^4K_{11/2}$	231.03588	[Rn]$5f^2 6d7s^2$	5.89	Actinides	
92	U	Uranium	$^5L^\circ_6$	238.02891	[Rn]$5f^3 6d7s^2$	6.1941	Actinides	
93	Np	Neptunium	$^6L_{11/2}$	(237)	[Rn]$5f^4 6d7s^2$	6.2655	Actinides	
94	Pu	Plutonium	$^7F_0$	(244)	[Rn]$5f^6 7s^2$	6.0258	Actinides	
95	Am	Americium	$^8S^\circ_{7/2}$	(243)	[Rn]$5f^7 7s^2$	5.9738	Actinides	
96	Cm	Curium	$^9D^\circ_2$	(247)	[Rn]$5f^7 6d7s^2$	5.9914	Actinides	
97	Bk	Berkelium	$^6H^\circ_{15/2}$	(247)	[Rn]$5f^9 7s^2$	6.1978	Actinides	
98	Cf	Californium	$^5I_8$	(251)	[Rn]$5f^{10} 7s^2$	6.2817	Actinides	
99	Es	Einsteinium	$^4I^\circ_{15/2}$	(252)	[Rn]$5f^{11} 7s^2$	6.3676	Actinides	
100	Fm	Fermium	$^3H_6$	(257)	[Rn]$5f^{12} 7s^2$	6.50	Actinides	
101	Md	Mendelevium	$^2F^\circ_{7/2}$	(258)	[Rn]$5f^{13} 7s^2$	6.58	Actinides	
102	No	Nobelium	$^1S_0$	(259)	[Rn]$5f^{14} 7s^2$	6.65	Actinides	
103	Lr	Lawrencium	$^2P^\circ_{1/2}$	(262)	[Rn]$5f^{14} 7s^2 7p$	4.90	Actinides	

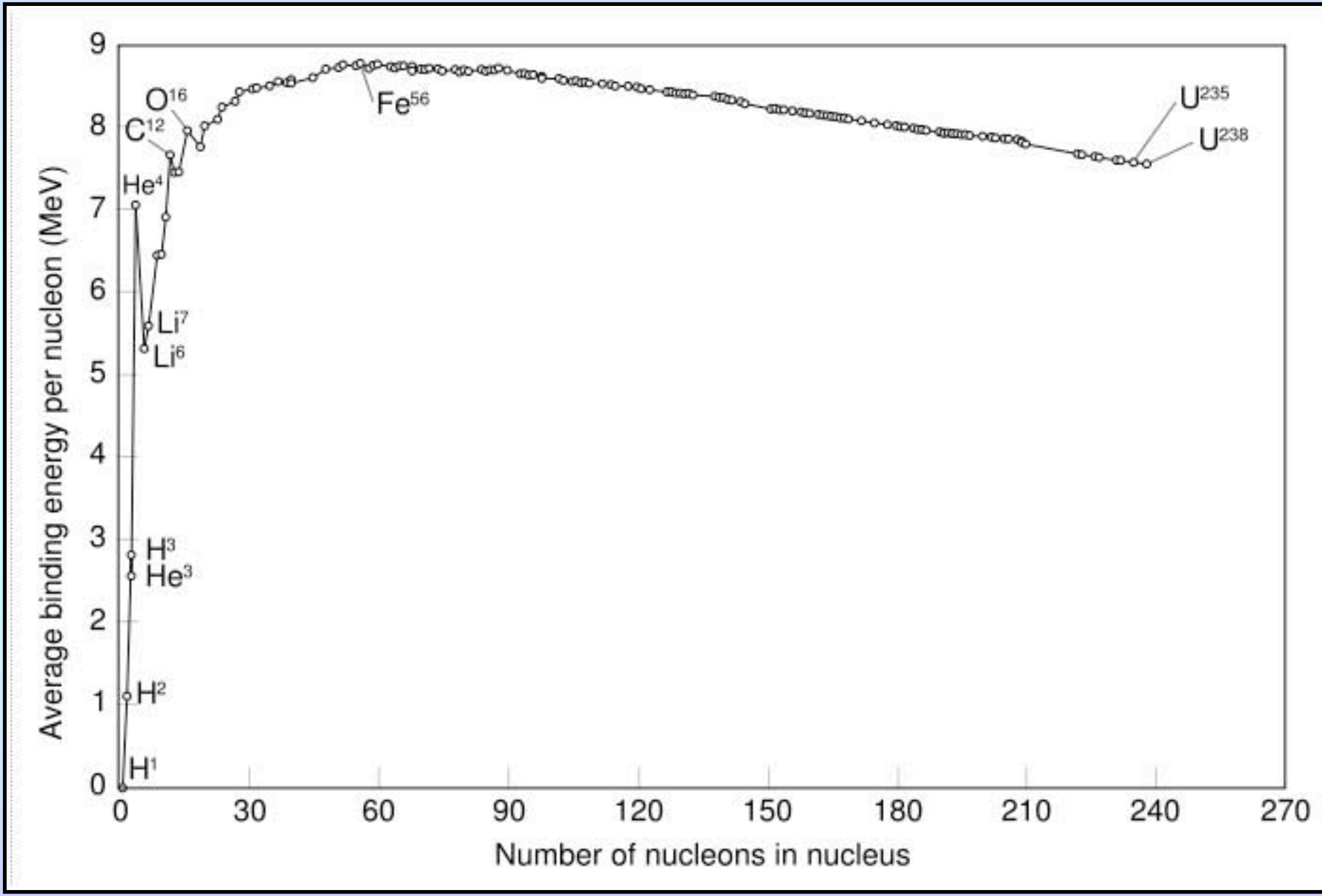
[†]Based upon ^{12}C. () indicates the mass number of the longest-lived isotope.

*IUPAC conventional atomic weights; standard atomic weights for these elements are expressed in intervals; see iupac.org for an explanation and values.

For a description of the data, visit physics.nist.gov/data

NIST SP 966 (September 2014)

Summary of Nuclear Binding Energies



Nucleosynthesis

Various elements/isotopes are made via different processes:

• ^{4}He	Hydrogen burning
• ^{2}H, Li, Be, B	Non-thermal processes (spallation)
• ^{14}N, ^{13}C, ^{15}N, ^{17}O	CNO processing
• ^{12}C, ^{16}O, ^{20}Ne	Helium burning
• ^{18}O, ^{22}Ne	α captures on ^{14}N (He burning)
• ^{20}Ne, Na, Mg, Al, ^{28}Si	Partly from carbon burning
• Mg, Al, Si, P, S	Partly from oxygen burning
• Ar, Ca, Ti, Cr, Fe, Ni	Partly from silicon burning
• Most other elements	Neutron captures

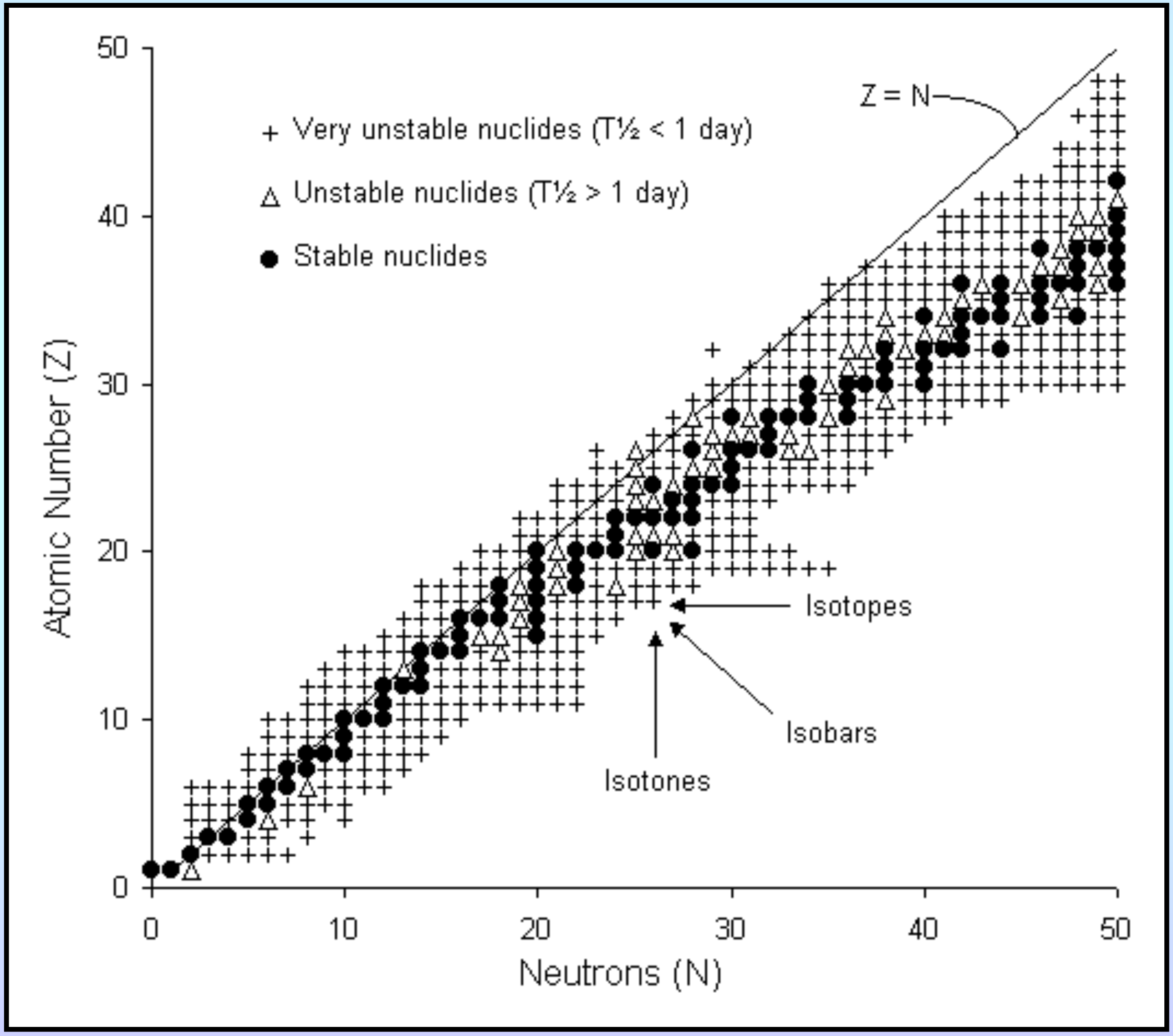
For many applications, these can be divided into 1) CNO processed elements, 2) α-process elements, 3) Fe-peak elements, and 4) neutron-capture elements.

Neutron-capture elements can further be divided into 2 (overlapping) groups: *s*-process elements and *r*-process elements.

s, *r*, and *p*-Process Elements

To understand the production of heavy elements, consider the locations of stable elements in a plot of proton number versus neutron number.

In general, there exists a "valley of stability" which contains most stable isotopes. On the edge of the valley are radioactive isotopes.

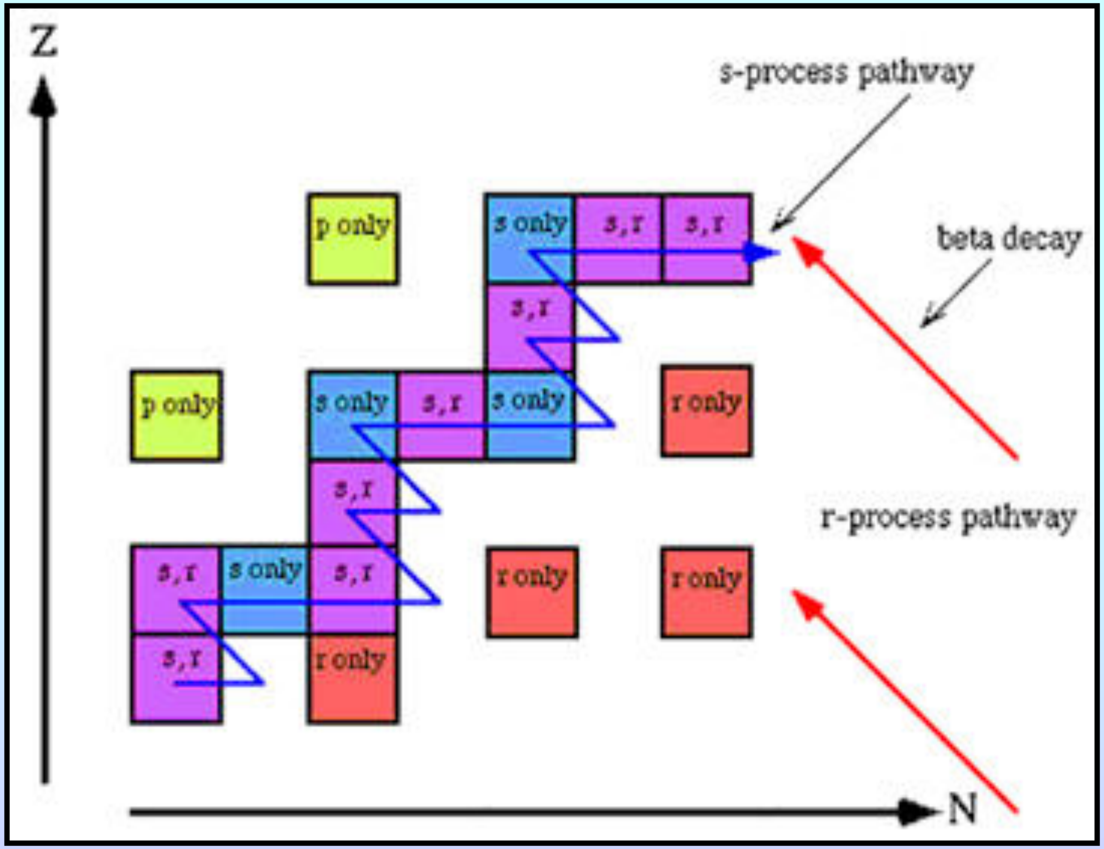


s, *r*, and *p*-Process Elements

s-process elements are created when neutrons are released <u>s</u>lowly during nucleosynthesis, so that the timescale for neutron capture is long compared to that for β-decay (which is typically hours to days). This generally begins during helium-shell burning with the reactions $^{13}C + {}^{4}He \rightarrow {}^{16}O + n$ and $^{22}Ne + {}^{4}He \rightarrow {}^{25}Mg + n$.

The *r*-process occurs when neutrons are released <u>r</u>apidly (i.e., during a supernova). Nuclei acquire a maximum number of neutrons, then decay later.

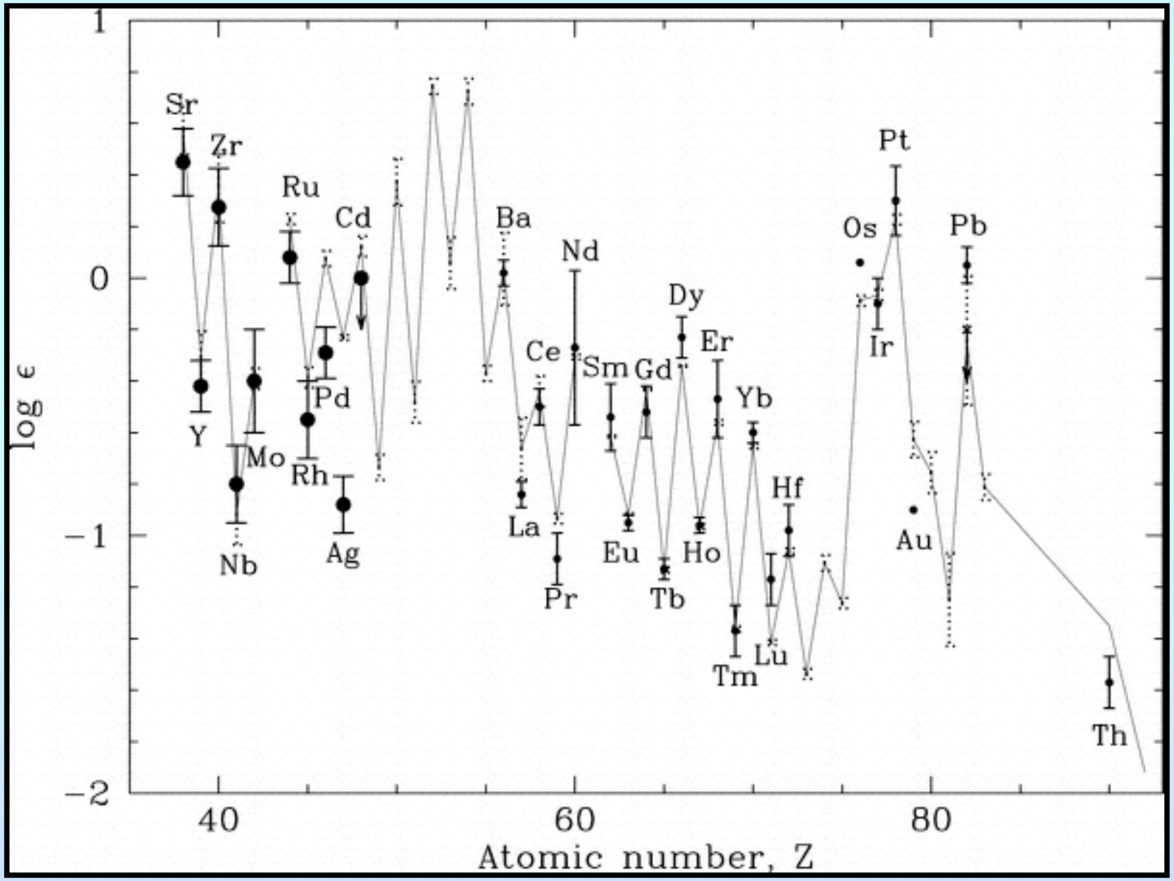
p-process elements are rarer, since <u>p</u>roton captures must overcome electrostatic repulsion.



Elemental Abundances

Since β-decay rates and neutron capture cross sections are well-known, the relative rates of production of s-process elements are well-known, i.e.,

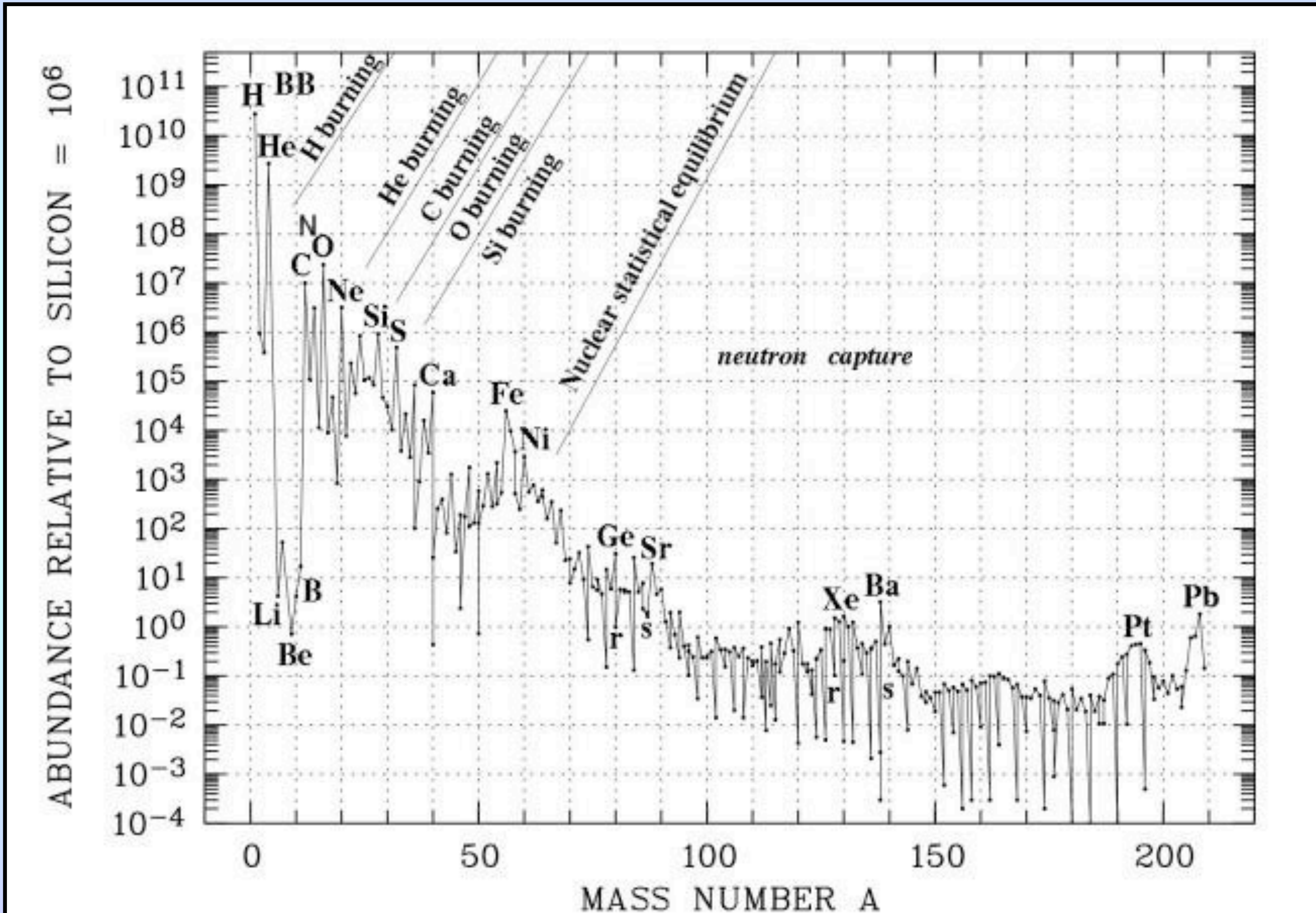
$$\frac{dN_A}{dt} = -\sigma_A N_A + \sigma_{A-1} N_{A-1}$$



Comparison of the theoretical abundance pattern for s-process only elements versus the observed abundances of a star.

What remains is due to the r-process.

“Universal” Elemental Abundances



Pulsating Stars

Every star in the HR diagram has a natural (fundamental) pulsation frequency. To see this, assume that about ½ of the star's time is spent in the expansion phase, and the other half in the contraction phase. Also assume that the star's change in radius is some percentage of its total radius, i.e., $\delta R \propto R$, and that the change in radius is mostly happening in the stars atmosphere. Gravity will restore the pulsation in a freefall timescale,

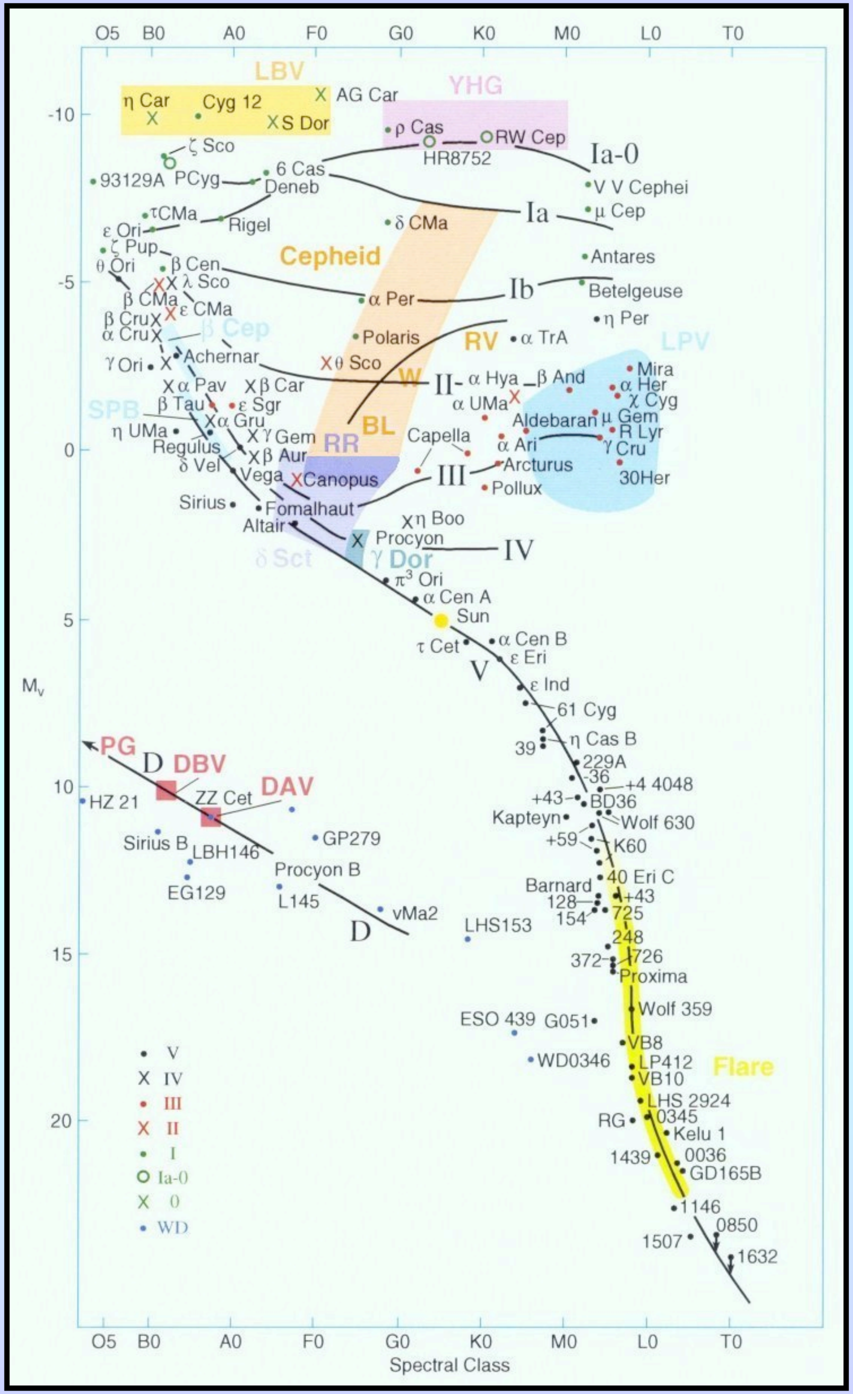
$$\frac{1}{2} g \tau_{ff}^2 = \frac{1}{2}\left(\frac{GM}{R^2}\right)\tau_{ff}^2 \propto R \quad \Rightarrow \quad \tau_{ff} \propto \left(\frac{R^3}{M}\right)^{1/2} \propto \langle \rho \rangle^{-1/2}$$

Since this is ~ ½ the cycle, the period of the star $P <\rho>^{1/2} = Q$. In fact, this simple equation is good for all stars; Q only varies by a couple of percent. But which stars pulsate?

Pulsating Stars

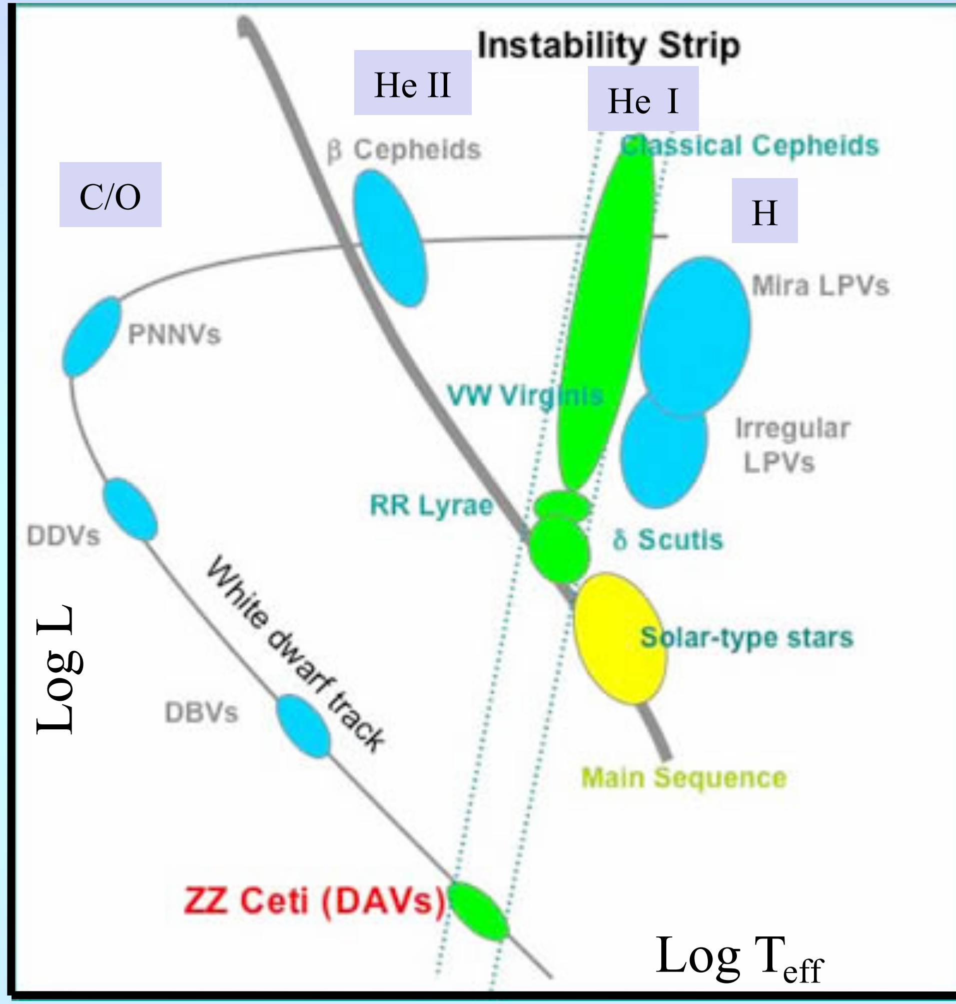
Consider a region in a star where a common species (H, He^0, He^+, etc.) is partially ionized. Compression increases the pressure and temperature, which increases the ionization fraction. The extra electrons increase the opacity, and the dammed-up energy causes expansion. This expansion reduces the temperature, prompting recombination, which then lowers the opacity, allowing the energy to escape.

If the partial ionization zone is too close to the surface, the mass driving the pulsation is negligible. If it is too deep in the star, the pulsation is damped out by the layers above. Hence the existence of “instability strips”.



Pulsating Stars

There are several areas of instability in the HR diagram. The classic "instability strip" which contains RR Lyr stars and Cepheid variables is due to the partial ionization of He I. Long period and Mira-like variables are driven by the partial ionization of hydrogen. Evolved stars (such as white dwarfs and planetary nebula nuclei) have so little H and He that C/O can drive pulsations.



Note: not all variables pulsate in their fundamental mode; some pulsate in overtones. On occasion, it's hard to tell the difference.

Pulsating Stars

Note that pulsating stars are extremely useful for astrophysics. If we substitute using $L = 4 \pi R^2 \sigma T^4$, then

$$P \propto \langle \rho \rangle^{-1/2} \propto \left(\frac{R^3}{M} \right)^{1/2} \propto \frac{R^{3/2}}{M^{1/2}} \propto \frac{L^{3/4}}{T_{\rm eff}^3 M^{1/2}}$$

If the instability strip is narrow, the temperature dependence can be neglected. Moreover, if a mass-luminosity relation ($L \propto M^\alpha$) exists, such as that for blue-loop stars, then

$$\log L = \log P + \mathrm{C}$$

The (easy to measure) period therefore defines the luminosity.